MATH CP
PAPER 1

福建中學

FUKIEN SECONDARY SCHOOL

S6 MOCK EXAMINATION (2021-2022)

MATHEMATICS Compulsory Part

PAPER 1

Question-Answer Book

Date : 14th January 2022
Time : 8:30 a.m. - 10:45 a.m.
(2 hours 15 minutes)

This paper must be answered in English.

INSTRUCTIONS

1. Write your Name, Class and Class Number in the spaces provided on Page 1.
2. This paper consists of THREE sections, A(1), A(2) and B.
3. Attempt ALL questions in this paper. Write your answers in the spaces provided in this Question-Answer Book. Do not write in the margins. Answers written in the margins will not be marked.
4. Graph paper and supplementary answer sheets will be supplied on request. Write your Name and mark the question number on each sheet.
5. Unless otherwise specified, all workings must be clearly shown.
6. Unless otherwise specified, numerical answers should be either exact or correct to 3 significant figures.
7. The diagrams in this paper are not necessarily drawn to scale.

Name	
Class	( )

	Marker's Use Only	
Question No.	Marks	
1–2		
3–4		
5–6		
7		
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9		
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11		
12		
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18		
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Total		

SECTION A(1) (35 marks)

1. Simplify $\dfrac{(-x^{-4})^2}{xy^{-1}}$ and express your answer with positive indices. (3 marks)

2. Make x the subject of the formula $\dfrac{y-2x}{2}+1=3y$. (3 marks)

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3. Factorize

(a) $4a^2 - 4ab + b^2$,

(b) $1 - 4a^2 + 4ab - b^2$.

(3 marks)

4. Consider the compound inequality

$$-6x \geq 12 \quad \text{and} \quad \frac{2x+5}{3} > 2(x+1) \qquad \ldots\ldots(*)$$

(a) Solve (*).

(b) Write down the greatest negative integer satisfying (*).

(4 marks)

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5. The total price of an apple and 3 oranges is $20 while the total price of an orange and 2 apples is $18. Find the total price of 2 apples and 3 oranges.

(4 marks)

6. The cost of a dress is $160. The marked price of the dress is 25% above its cost.
   (a) Find the marked price of the dress.
   (b) If the dress is sold at a price such that there is no loss or gain, find the discount per cent on the marked price of the dress.

(4 marks)

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7. In a polar coordinate system, O is the pole. The polar coordinates of points A and B are (6, 35°) and (6, 155°) respectively. Let L be the line which bisects $\angle AOB$.

(a) Is L perpendicular to AB? Explain your answer.

(b) Find the polar coordinates of the point of intersection of L and AB.

(5 marks)

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8. In Figure 1, *BDC* is a straight line. *AC* and *DE* intersect at the point *F*. It is given that $AC \perp DE$ and *AC* is the angle bisector of $\angle DAE$.

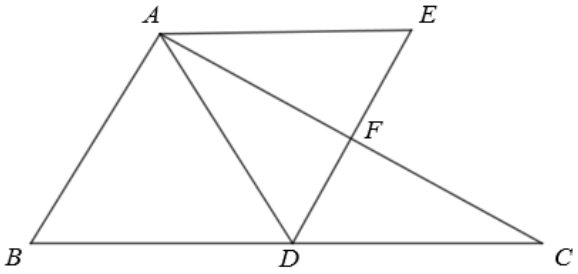

Figure 1

(a) Prove that $\triangle AFD \cong \triangle AFE$.

(b) It is given that *AD* is a median of $\triangle ABC$ and $AE = DC$. Is $\triangle ABC$ a right-angled triangle? Explain your answer.

(5 marks)

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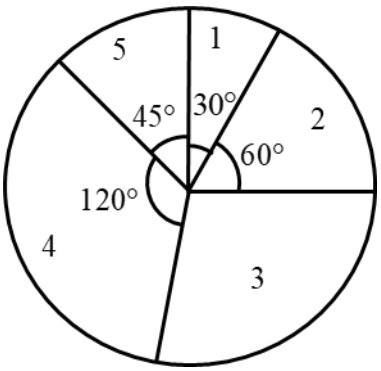
9. The pie chart below shows the distribution of the grades of a group of students in an aptitude test.



Distribution of the grades of the group of students in the aptitude test

(a) Write down the mean and the mode of the distribution.

(b) A student is randomly selected from the group. Find the probability that the grade of the selected student is higher than 2.

(4 marks)

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SECTION A(2) (35 marks)

10. In a company, the number of magazines printed is n and the profit of selling magazines is \$$P$. It is given that P is the sum of two parts, one part varies as n and the other part varies as the square of n. When $n = 3000$, $P = 11\ 325$ and when $n = 6000$, $P = 45\ 150$.

(a) Find the profit when 5500 magazines are printed. (3 marks)

(b) If the profit is \$64 980, find the number of magazines printed. (2 marks)

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11. The stem-and-leaf diagram below shows the distribution of the time (in minutes) spent on lunch by 6A students on a specific day.

Stem (10 minutes)	Leaf (1 minute)
0	8 9
1	2 *a* *a* *a* 7
2	0 1 3 5 6 8
3	0 2 *b* 8
4	3 8 9

It is given that the median of the above distribution is greater than the mode by 9 minutes.

(a) Find a. (2 marks)

(b) It is given that the difference between the range and the inter-quartile range of the distribution is less than 22 minutes. Find

(i) b,

(ii) the greatest possible standard deviation of the distribution.

(4 marks)

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12. Figure 2(a) shows a closed vessel with some water, which is in the shape of an inverted right circular cone. The base radius and the height of the cone are 18 cm and 24 cm respectively. Kelly finds that the depth of water in the vessel is 8 cm.

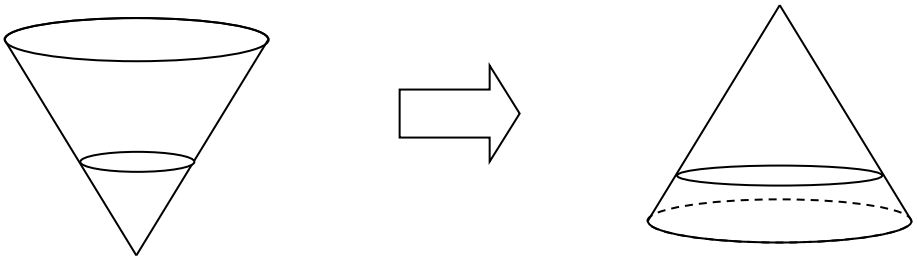

Figure 2(a) Figure 2(b)

(a) (i) Find the radius of the water surface.

(ii) Find the area of the wet curved surface of the vessel in terms of π.

(3 marks)

(b) Kelly turns the vessel upside down as shown in Figure 2(b).

(i) Find the new depth of water, correct to 1 decimal place.

(ii) Kelly claims that the wet curved surface of the vessel has increased. Do you agree? Explain your answer.

(5 marks)

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13. Let $f(x) = 12x^4 + 56x^3 + px^2 + qx + r$, where p, q and r are constants. It is given that $f(1) - f(-1) = 102$ and $f(x) = (ax^2 + x + b)(2x^2 + cx + 4)$, where a, b and c are constants.

(a) Find a, b and c. (5 marks)

(b) How many distinct real root(s) does the equation $f(x) = 0$ have? Explain your answer. (2 marks)

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14. The coordinates of points A and B are (2, 16) and (–4, 18) respectively. The circle C passes through A and B and intersects the x-axis at the points P and Q. It is given that $\angle ABP = 90°$.

(a) Find the coordinates of P. (2 marks)

(b) Find the equation of C. (3 marks)

(c) Let R be a moving point in the rectangular coordinate plane such that R is equidistant from P and Q. Denote the locus of R by Γ.

(i) Describe the geometric relationship between Γ and PQ.

(ii) Suppose that Γ cuts the x-axis at point D. Someone claims that the area of $\triangle ADQ$ is the same as the area of $\triangle ABQ$. Do you agree? Explain your answer.

(4 marks)

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SECTION B (35 marks)

15. 4 boys and 8 girls are randomly arranged to sit in a row.

(a) Find the probability that all the boys are sitting together. (2 marks)

(b) Find the probability that none of the boys is sitting next to each other. (2 marks)

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16. Chris and John are two salesmen in a company and their difference in salaries is $3000. The standard scores of the salaries of Chris and John are 1 and –2 respectively. Find the variance of the distribution of the salaries. (3 marks)

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17. Let α_n and β_n be the roots of the quadratic equation $x^2 - \sqrt{\log_3 2^n}\,x + \log_9 \sqrt{2^n} = 0$, where n is a positive integer.

(a) Express $\alpha_n{}^2 + \beta_n{}^2$ in terms of n. (3 marks)

(b) Given that $\alpha_1{}^2 + \beta_1{}^2 + \alpha_2{}^2 + \beta_2{}^2 + ... + \alpha_k{}^2 + \beta_k{}^2 = \log_3 32^k$, find k. (3 marks)

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18. In Figure 3(a), *ABCD* is a paper card. It is given that *ACD* is an equilateral triangle of side 24 cm, *BC* = 23 cm and $\angle ABC = 70°$.

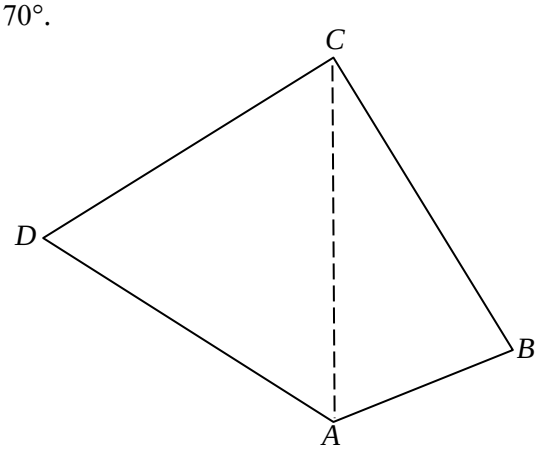

Figure 3(a)

(a) Find

(i) $\angle BAC$,

(ii) the shortest distance from *B* to *AC*. (4 marks)

(b) The paper card in Figure 3(a) is folded along *AC* such that *AB* and *AD* lie on the horizontal ground as shown in Figure 3(b). It is given that $\angle BAD = 45°$.

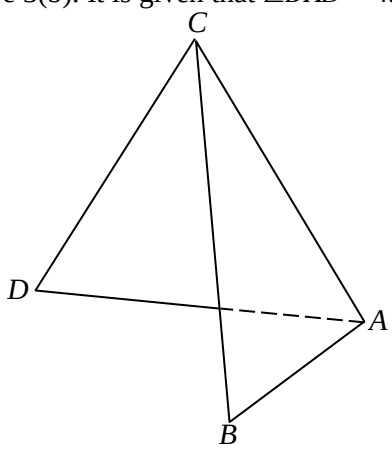

Figure 3(b)

Find

(i) the angle between the face *ABC* and the face *ACD*,

(ii) the area of $\triangle ABD$,

(iii) the shortest distance from *C* to the horizontal ground. (7 marks)

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19. Let $f(x) = x^2 - 8kx - 6x + 16k^2 + 22k + 5$, where k is a constant. Let M be the vertex of the graph of $y = f(x)$.

(a) Using the method of completing the square, express the coordinates of M in terms of k.

(2 marks)

(b) Let N be the vertex of the graph of $y = -f(x - 6)$.

(i) Write down the coordinates of N in terms of k.

(ii) Let P be a moving point on the rectangular coordinate plane such that P is the orthocentre of $\triangle PMN$. Denote the locus of P by Γ. Express the equation of Γ in terms of k.

(iii) Let U and V be two points on the coordinate plane such that $MUNV$ is a square. Express the equation of UV in terms of k.

(iv) Someone claims that the area of the square $MUNV$ cannot be less than 18 square units. Do you agree? Explain your answer.

(9 marks)

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