福建中學

FUKIEN SECONDARY SCHOOL

S6 Mock Examination (2021-2022)
Mathematics Extended Part Module 1
(2 hours 30 minutes)

Date: 26th January 2022
Time: 8:30 a.m. – 11:00 a.m.

Name:______________
Class:______ No. :____

Instructions to students:

1. Write your name, class and class number in the spaces provided on this cover.

2. This paper consists of TWO sections, A and B. Section A carries 50 marks. Section B carries 50 marks.

3. Attempt ALL questions in Sections A and B. Write your answers in the spaces provided in the Question-Answer Book. Do not write in the margins. Answers written in the margins will not be marked.

4. The Question-Answer book will be collected at the end of the examination.

5. Unless otherwise specified, show all workings clearly.

6. Unless otherwise specified, numerical answers should be either exact or given to 4 decimal places.

Section A(50 marks)

1. The table below shows the probability distribution of a discrete random variable X, where h and k are constants:

x	-3	-1	2	4	7
$P(X=x)$	h	0.26	0.47	k	0.03

It is given that $E(X) = 1.22$. Find

(a) h and k,

(b) $E((3X-5)^2)$ and $\mathrm{Var}(3X-5)$.

(6 marks)

2. Let A and B be two events. Denote the complementary events of A and B by A' and B' respectively. Suppose that $P(B \mid A') = 0.4$, $P(A' \mid B') = 1.8P(B)$ and $P(A' \cup B') = 1.288 - P(B)$.

(a) Prove that $P(A') = 3P(B)P(B')$.

(b) Find $P(A')$.

(c) Are A and B independent? Explain your answer.

(6 marks)

3. A puzzle competition consists of two stages, I and II. The probability that a player wins in each stage is 0.8. If a player does not win in stage I, he/she cannot get any gifts. If a player wins in stage I, he/she will be promoted to stage II. There is a lucky draw after stage II. If a player wins in stage II, the probability that he/she gets a gift in the lucky draw is 0.8. If a player does not win in stage II, the probability that he/she gets a gift in the lucky draw is 0.1.
   (a) Find the probability that a player cannot get any gifts.
   (b) Given that a player cannot get any gifts, find the probability that he/she does not win in stage II.
   (c) Find the probability that exactly 4 out of 7 players cannot get any gifts.

(6 marks)

4. The time taken by Brian to swim 50 m follows a normal distribution with a mean of 48 seconds and a standard deviation of σ seconds. If a time record of Brian to swim 50 m is selected at random, the probability for the time taken in the record lying between 45 seconds and 51 seconds is 0.7698.
   (a) Find σ.
   (b) Suppose a sample of 16 time records of Brian to swim 50 m is selected at random. Find the probability that the mean time taken in the sample exceeds 48.9 seconds.
   (c) Another sample of n time records of Brian to swim 50 m is selected at random such that the probability for the mean time taken in the sample exceeding 47.4 seconds is higher than 0.95. Find the least value of n.

(7 marks)

5. Let c be a constant.

(a) Expand $(3-e^{cx})^2$ in ascending powers of x as far as the term in x^2.

(b) If the coefficient of x^2 is twice the coefficient of x in the expansion of $(3-e^{cx})^2(1-x)^8$, find c.

(6 marks)

6. Consider the curve C: $y=\dfrac{2x}{\sqrt{3x^2+1}}$.

(a) Find $\dfrac{dy}{dx}$.

(b) A tangent to C passes through the point $(-3, 0)$. Find the slope of this tangent.

(6 marks)

7. Define $f(x) = \dfrac{3x-6}{x+4}$ for all $x > -4$.

(a) Prove that $f(x)$ is increasing.

(b) Find $\lim\limits_{x \to \infty} f(x)$.

(c) Find the exact value of the area of the region bounded by the graph of $y = f(x)$, the x-axis and the y-axis.

(6 marks)

8. (a) By considering $\frac{d}{dx}[(2x^2+3x)e^{-2x}]$, find $\int (2x^2+x)e^{-2x}dx$.

(b) Define $f(x)=(4x^2+2x+1)e^{-2x}$ for all real numbers x. Let α and β be the two roots of the equation $f'(x)=0$, where $\alpha>\beta$. Find α and β. Also express $\int_\beta^\alpha f(x)\,dx$ in terms of e.

(7 marks)

Section B(50 marks)

9. In a certain challenge, a candidate has to compete with a computer in a series of tasks. The number of tasks won by the candidate follows a Poisson distribution with a mean of 2 per challenge. The points scored by the candidate in a task follows a normal distribution with a mean of 78.5 points and a standard deviation of 6 points.

   (a) Find the probability that the candidate wins fewer than 7 tasks in a certain challenge.

   (3 marks)

   (b) Find the probability that the candidate scores higher than 80 points in a certain task.

   (2 marks)

   (c) The candidate receives a token if the candidate wins a task and scores more than 80 points. The candidate is awarded a prize in a certain challenge if the candidate receives more than 3 tokens in that challenge.

   (i) Find the probability that the candidate wins exactly 4 tasks in a certain challenge and is awarded a prize in that challenge.

   (ii) If the candidate wins exactly 5 tasks in a certain challenge, find the probability that the candidate is awarded a prize in that challenge.

   (iii) Given that the candidate wins fewer than 7 tasks in a certain challenge, find the probability that the candidate is awarded a prize in that challenge.

   (7 marks)

10. A lunch supplier provides lunch for school *A*. Students in school *A* complain that the quality of food is too bad. Therefore, the principal of school *A* decides to ask for the evaluation of the lunch by survey (0 is the lowest, 100 is the highest). From past experience, the marks of a randomly selected student follow a normal distribution with mean μ and a standard deviation 16.

(a) Teachers of school *A* use the marks collected to estimate μ.

(i) A random sample of 16 students is taken and their marks are recorded as below:

20	25	25	30	35	35	35	40
45	60	60	65	65	80	80	95

Construct a 95% confidence interval for μ.

(ii) Find the least sample size to be taken such that the width of the 95% confidence interval for μ is less than 8 marks.

(7 marks)

(b) Suppose that $\mu = 50$. The lunch supplier will give a cash refund to students whose result in the survey is less than 42 marks.

(i) Find the probability that the lunch supplier gives more than 2 cash refunds to the first 8 students interviewed.

(ii) Find the probability that the 4th cash refund is given to the 15th student interviewed.

(6 marks)

11. The citizens in town A suffer from a disease. The number of citizens of town A (in thousands) is modelled by

$$N = \frac{32}{4 + \alpha t e^{\beta t}},$$

where $t(\geq 0)$ is the number of days after the disease has begun, α and β are constants.

(a) Express $\ln\left(\frac{32-4N}{Nt}\right)$ as a linear function of t.

(2 marks)

(b) It is given that the intercept on the horizontal axis of the graph of the linear function and the slope obtained in (a) are $2\ln 0.05$ and -0.5 respectively.

(i) Find α and β.

(ii) Will the number of citizens in town A be less than 8 thousand on a certain day after the disease has begun? Explain your answer.

(iii) Describe how the rate of change of the number of citizens in town A varies during the first 4 days after the disease has begun. Explain your answer.

(10 marks)

12. A company launches a new product. The rate of change of the operating cost of the company (in million dollars per month) can be modelled by

$$A(t) = \ln\sqrt{4t^2 + 12t + 15},$$

where $t (0 \le t \le 6)$ is the number of months elapsed since the launch of the new product. Denote the total operating cost of the company in the first 2 months since the launch of the new product by α million dollars. Let α_1 be the estimate of α by using the trapezoidal rule with 4 sub-intervals.

(a) (i) Find α_1.

(ii) Find $A''(t)$.

(5 marks)

(b) The company models the rate of change of the revenue of the company (in million dollars per month) by

$$B(t) = \frac{10e^{\frac{3}{2}t}}{1 + e^{\frac{3}{2}t}},$$

where $t (0 \le t \le 6)$ is the number of months elapsed since the launch of the new product. Denote the total profit made by the company in the first 2 months since the launch of the new product by β million dollars.

(i) Find $\int_0^2 B(t)dt$.

(ii) Using the results of (a)(i) and (b)(i), estimate β.

(iii) The company claims that $\frac{\beta}{\alpha} < 3.7$. Do you agree? Explain your answer.

(8 marks)

END OF PAPER

Standard Normal Distribution Table

z	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
0.0	.0000	.0040	.0080	.0120	.0160	.0199	.0239	.0279	.0319	.0359
0.1	.0398	.0438	.0478	.0517	.0557	.0596	.0636	.0675	.0714	.0753
0.2	.0793	.0832	.0871	.0910	.0948	.0987	.1026	.1064	.1103	.1141
0.3	.1179	.1217	.1255	.1293	.1331	.1368	.1406	.1443	.1480	.1517
0.4	.1554	.1591	.1628	.1664	.1700	.1736	.1772	.1808	.1844	.1879
0.5	.1915	.1950	.1985	.2019	.2054	.2088	.2123	.2157	.2190	.2224
0.6	.2257	.2291	.2324	.2357	.2389	.2422	.2454	.2486	.2517	.2549
0.7	.2580	.2611	.2642	.2673	.2704	.2734	.2764	.2794	.2823	.2852
0.8	.2881	.2910	.2939	.2967	.2995	.3023	.3051	.3078	.3106	.3133
0.9	.3159	.3186	.3212	.3238	.3264	.3289	.3315	.3340	.3365	.3389
1.0	.3413	.3438	.3461	.3485	.3508	.3531	.3554	.3577	.3599	.3621
1.1	.3643	.3665	.3686	.3708	.3729	.3749	.3770	.3790	.3810	.3830
1.2	.3849	.3869	.3888	.3907	.3925	.3944	.3962	.3980	.3997	.4015
1.3	.4032	.4049	.4066	.4082	.4099	.4115	.4131	.4147	.4162	.4177
1.4	.4192	.4207	.4222	.4236	.4251	.4265	.4279	.4292	.4306	.4319
1.5	.4332	.4345	.4357	.4370	.4382	.4394	.4406	.4418	.4429	.4441
1.6	.4452	.4463	.4474	.4484	.4495	.4505	.4515	.4525	.4535	.4545
1.7	.4554	.4564	.4573	.4582	.4591	.4599	.4608	.4616	.4625	.4633
1.8	.4641	.4649	.4656	.4664	.4671	.4678	.4686	.4693	.4699	.4706
1.9	.4713	.4719	.4726	.4732	.4738	.4744	.4750	.4756	.4761	.4767
2.0	.4772	.4778	.4783	.4788	.4793	.4798	.4803	.4808	.4812	.4817
2.1	.4821	.4826	.4830	.4834	.4838	.4842	.4846	.4850	.4854	.4857
2.2	.4861	.4864	.4868	.4871	.4875	.4878	.4881	.4884	.4887	.4890
2.3	.4893	.4896	.4898	.4901	.4904	.4906	.4909	.4911	.4913	.4916
2.4	.4918	.4920	.4922	.4925	.4927	.4929	.4931	.4932	.4934	.4936
2.5	.4938	.4940	.4941	.4943	.4945	.4946	.4948	.4949	.4951	.4952
2.6	.4953	.4955	.4956	.4957	.4959	.4960	.4961	.4962	.4963	.4964
2.7	.4965	.4966	.4967	.4968	.4969	.4970	.4971	.4972	.4973	.4974
2.8	.4974	.4975	.4976	.4977	.4977	.4978	.4979	.4979	.4980	.4981
2.9	.4981	.4982	.4982	.4983	.4984	.4984	.4985	.4985	.4986	.4986
3.0	.4987	.4987	.4987	.4988	.4988	.4989	.4989	.4989	.4990	.4990
3.1	.4990	.4991	.4991	.4991	.4992	.4992	.4992	.4992	.4993	.4993
3.2	.4993	.4993	.4994	.4994	.4994	.4994	.4994	.4995	.4995	.4995
3.3	.4995	.4995	.4995	.4996	.4996	.4996	.4996	.4996	.4996	.4997
3.4	.4997	.4997	.4997	.4997	.4997	.4997	.4997	.4997	.4997	.4998
3.5	.4998	.4998	.4998	.4998	.4998	.4998	.4998	.4998	.4998	.4998

Note : An entry in the table is the area under the standard normal curve between $x = 0$ and $x = z$ ($z \geq 0$) .
Areas for negative values of z can be obtained by symmetry.