FUKIEN SECONDARY SCHOOL

S6 First Term Uniform Test (2021-2022)

Mathematics Extended Part Module 2

(1 hour 15 minutes)

Date: 11th November 2021 Name: ___________________

Time: 10:15a.m. - 11:30a.m. Class:________ No.:_______

Instructions to students:

1. This paper consists of TWO sections, A and B. The full score is 53.
2. Attempt ALL questions in Section A and Section B.
3. Write your answers in the spaces provided.
4. Unless otherwise specified, show your workings clearly.
5. Unless otherwise specified, numerical answers must be exact.
6. The diagrams in this paper are not necessarily drawn to scale.

FORMULAS FOR REFERENCE

$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$	$\sin A + \sin B = 2\sin\frac{A+B}{2}\cos\frac{A-B}{2}$
$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$	$\sin A - \sin B = 2\cos\frac{A+B}{2}\sin\frac{A-B}{2}$
$\tan(A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A \tan B}$	$\cos A + \cos B = 2\cos\frac{A+B}{2}\cos\frac{A-B}{2}$
$2\sin A \cos B = \sin(A+B) + \sin(A-B)$	$\cos A - \cos B = -2\sin\frac{A+B}{2}\sin\frac{A-B}{2}$
$2\cos A \cos B = \cos(A+B) + \cos(A-B)$	
$2\sin A \sin B = \cos(A-B) - \cos(A+B)$	

Section A (28 marks)

1. Let $q(x) = \dfrac{3x}{2x^2 - 5}$. Prove that $q(1 + h) - q(1) = \dfrac{2h^2 + 7h}{2h^2 + 4h - 3}$. Hence, find $q'(1)$ from first principles.

(4 marks)

2. (a) Using integration by parts, find $\int x^2 e^{4x} dx$.

(b) Let R be the region bounded by the curve $y = 8xe^{2x}$, the straight line $x = 1$ and the x-axis. Find the volume of the solid of revolution generated by revolving R about the x-axis.

(6 marks)

3.

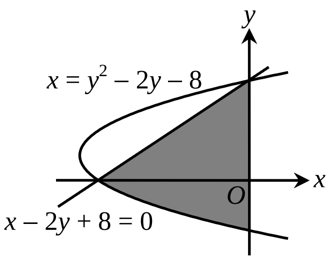

Figure 1

Find the area bounded by the curve $x = y^2 - 2y - 8$, the y-axis and the line $x - 2y + 8 = 0$ in Figure 1.

(5 marks)

4. In Figure 2, AC and BD intersect at E. It is given that $BD = (1+3r)ED$ and $\overrightarrow{AC} = 3\overrightarrow{AB} + 2\overrightarrow{AD}$.

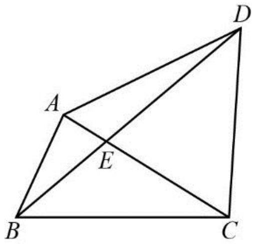

Figure 2

(a) Express $\overrightarrow{AE}$ in terms of $\overrightarrow{AB}$, $\overrightarrow{AD}$ and r.

(b) Find the value of r.

(5 marks)

5. Let $M = \begin{pmatrix} 1 & k \\ 0 & 1 \end{pmatrix}$, where k is a real number.

(a) (i) Express M^2 in terms of k.

(ii) Express M^n in terms of k and n, where n is a positive integer.

(b) Let $A = \begin{pmatrix} -15 & 36 \\ -9 & 21 \end{pmatrix}$ and $P = \begin{pmatrix} 2 & 5 \\ 1 & 3 \end{pmatrix}$.

(i) Find P^{-1}.

(ii) It is given that $P^{-1}AP = \begin{pmatrix} 3 & 9 \\ 0 & 3 \end{pmatrix}$. Find A^{16}.

(8 marks)

Section B (25 marks)

6. Consider the system of linear equations in real variables x, y, z

$$(E):\ \begin{cases} x+y-z=a \\ x-y-(\lambda-1)z=b \\ \lambda x+3y-2z=4a \end{cases},$$

where λ, a and b are real numbers.

(a) Assume that (E) has a unique solution.

(i) Find the range of values of λ.

(ii) Express z in terms of λ, a and b.

(5 marks)

(b) Assume that $\lambda = 1$ and (E) is consistent.

(i) Express b in terms of a.

(ii) Hence, solve (E) in terms of a.

(4 marks)

(c) If (x, y, z) is a real solution of the system of linear equations

$$\begin{cases} x+y-z=6 \\ x-y=-12 \\ x+3y-2z=24 \end{cases},$$

is $x^2 + y^2 + z^2 > 75$ always true? Explain your answer. (3 marks)

7. Let $\overrightarrow{OA} = \mathbf{i} - 2\mathbf{j}$, $\overrightarrow{OB} = -7\mathbf{i} - 2\mathbf{j} + 4\mathbf{k}$ and $\overrightarrow{OC} = -\mathbf{i} + \mathbf{j} + 2\mathbf{k}$, where O is the origin.

(a) (i) Find $\overrightarrow{AB} \times \overrightarrow{AC}$.

(ii) Hence, find the volume of the tetrahedron $OABC$.

(5 marks)

(b) Denote the plane containing A, B and C by Π. Let $\overrightarrow{OP} = -4\mathbf{j} + 8\mathbf{k}$ and $\overrightarrow{OQ} = -9\mathbf{i} + 2\mathbf{j} - 10\mathbf{k}$.

The straight line passing through P and Q intersects Π at D.

(i) Is $\overrightarrow{PQ}$ perpendicular to Π? Explain your answer.

(ii) Find $PD : DQ$.

(iii) Describe the geometric relationship between AB, CD and PQ. Explain your answer.

(8 marks)

End of Test