FUKIEN SECONDARY SCHOOL

S6 First Term Uniform Test (2021-2022)

Mathematics Extended Part Module 1

(1 hour 15 minutes)

Date: 9th November 2021 Name:____________________

Time: 10:30 a.m. - 11:45 a.m. Class: ________ No.: _______

Instructions to students:

1. This paper consists of Section A (32 marks) and Section B(26 marks).

2. Attempt ALL questions. Write your answers on the single-lined paper.

3. Unless otherwise specified, show your workings clearly.

4. Unless otherwise specified, numerical answers should be either exact or given to 4 decimal places.

5. The diagrams in this paper are not necessarily drawn to scale.

Section A (32 marks)

1. The table below shows the probability distribution of a discrete random variable X, where a and b are constants:

x	1	4	9	16	25
$P(X=x)$	0.12	a	0.15	0.29	b

It is given that E(X) = 9.34. Find

(a) a and b,

(b) $E[(3-2X)^2]$ and Var(3 – 2X).

(6 marks)

2. Let A and B be two events. Denote the complementary events of A and B by A' and B' respectively. Suppose that $P(A \cap B) = 0.07$ and $P(B' \mid A) = 3P(A')$.

(a) Find $P(A)$.

(b) If A and B are independent, find $P(A \cup B)$.

(6 marks)

3. Let $f(x)$ be a continuous function such that $f'(x) = \dfrac{36-18x}{(2x^2-8x+11)^3}$ for all real numbers x.

(a) If $f(x)$ attains its maximum value at $x = a$, find a.

(b) It is given that the extreme value of $f(x)$ is 3. Find

(i) $f(x)$,

(ii) $\lim_{x\to\infty} f(x)$.

(6 marks)

5. Let $f(x) = px^3 - 6x^2 + qx + 63$, where p and q are constants. Denote the curve of $y = f(x)$ by Γ. It is given that (3, 225) is a turning point of Γ.

(a) Find p and q.

(b) Find the minimum value(s) and the maximum value(s) of $f(x)$.

(c) Write down the equation(s) of the horizontal tangent(s) to Γ.

(7 marks)

8. Define $f(x) = \dfrac{\sqrt{\ln x} - 2}{x}$ for all $x \geq 1$.

(a) Using integration by substitution, find $\int f(x)dx$.

(b) Denote the curve $y = f(x)$ by C.

(i) Write down the x-intercept(s) of C.

(ii) Find the area of the region bounded by C, the x-axis and the straight lines $x = e$ and $x = e^9$.

(7 marks)

Section B (26 marks)

6. The duration X (in seconds) of an advertisement broadcasted on a certain TV channel follows a normal distribution with mean μ seconds and standard deviation σ seconds. It is known that 17.11% of the durations are longer than 17.8 seconds and 38.30% of the durations fall within 2 seconds of μ.

   (a) Find μ and σ.

   (4 marks)

   (b) If the duration of an advertisement is longer than h seconds, extra advertising fee will be charged. Suppose that a total of 600 advertisements are played on the TV channel in a day. Assuming independence among durations of different advertisements, find the minimum integral value of h such that the expected number of advertisements with extra advertising fee is at most 9 in that day.

   (3 marks)

   (c) The advertising fee per second is adjusted in order to shorten the duration of advertisements. After adjustment, the duration Y (in seconds) of an advertisement follows a normal distribution with mean θ seconds and standard deviation 3.6 seconds.

   (i) Manager P draws a random sample of 16 advertisements and the durations (in seconds) are recorded as follows:

12	16	10	15	15	16	9	18
13	11	14	16	8	12	13	10

   Construct a 95% confidence interval for θ.

   (ii) Manager Q is going to draw another random sample of n advertisements. He requires that the probability that the mean duration of the n advertisements falls within 2 seconds of θ be greater than 0.995. Find the minimum value of n to meet his requirement.

   (6 marks)

7. A tank contains some water. Water is now poured into the tank. Let V m^3 be the volume of water in the tank. It is given that

$$V = 57 - \frac{360}{ae^{bt} + 6},$$

where t (≥ 0) is the number of seconds elapsed since the pouring begins and a and b are constants.

(a) Express $\ln\left(\frac{360}{57-V} - 6\right)$ as a linear function of t.

(1 mark)

(b) It is given that the graph of the linear function obtained in (a) passes through the origin and the point (20, 2). Find

(i) a and b,

(ii) $\frac{dV}{dt}$,

(iii) the value of V when $\frac{dV}{dt}$ attains its greatest value.

(8 marks)

(c) Amy finds that $S = 3V^{\frac{2}{3}}$, where S m^2 is the wet total surface area of the tank.

(i) Find the value of $\frac{dS}{dt}$ when $\frac{dV}{dt}$ attains it greatest value.

(ii) Amy claims that $\frac{dS}{dt}$ attains its greatest value when $\frac{dV}{dt}$ attains its greatest value.

Is the claim correct? Explain your answer.

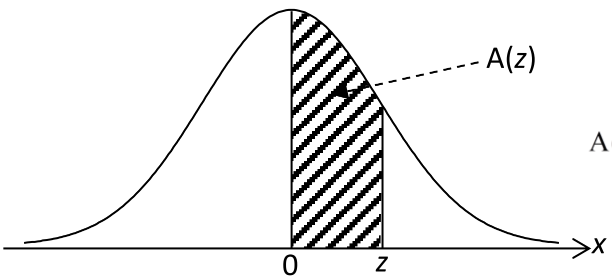
(4 marks)

End of Paper

Standard Normal Distribution Table

z	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
0.0	.0000	.0040	.0080	.0120	.0160	.0199	.0239	.0279	.0319	.0359
0.1	.0398	.0438	.0478	.0517	.0557	.0596	.0636	.0675	.0714	.0753
0.2	.0793	.0832	.0871	.0910	.0948	.0987	.1026	.1064	.1103	.1141
0.3	.1179	.1217	.1255	.1293	.1331	.1368	.1406	.1443	.1480	.1517
0.4	.1554	.1591	.1628	.1664	.1700	.1736	.1772	.1808	.1844	.1879
0.5	.1915	.1950	.1985	.2019	.2054	.2088	.2123	.2157	.2190	.2224
0.6	.2257	.2291	.2324	.2357	.2389	.2422	.2454	.2486	.2517	.2549
0.7	.2580	.2611	.2642	.2673	.2704	.2734	.2764	.2794	.2823	.2852
0.8	.2881	.2910	.2939	.2967	.2995	.3023	.3051	.3078	.3106	.3133
0.9	.3159	.3186	.3212	.3238	.3264	.3289	.3315	.3340	.3365	.3389
1.0	.3413	.3438	.3461	.3485	.3508	.3531	.3554	.3577	.3599	.3621
1.1	.3643	.3665	.3686	.3708	.3729	.3749	.3770	.3790	.3810	.3830
1.2	.3849	.3869	.3888	.3907	.3925	.3944	.3962	.3980	.3997	.4015
1.3	.4032	.4049	.4066	.4082	.4099	.4115	.4131	.4147	.4162	.4177
1.4	.4192	.4207	.4222	.4236	.4251	.4265	.4279	.4292	.4306	.4319
1.5	.4332	.4345	.4357	.4370	.4382	.4394	.4406	.4418	.4429	.4441
1.6	.4452	.4463	.4474	.4484	.4495	.4505	.4515	.4525	.4535	.4545
1.7	.4554	.4564	.4573	.4582	.4591	.4599	.4608	.4616	.4625	.4633
1.8	.4641	.4649	.4656	.4664	.4671	.4678	.4686	.4693	.4699	.4706
1.9	.4713	.4719	.4726	.4732	.4738	.4744	.4750	.4756	.4761	.4767
2.0	.4772	.4778	.4783	.4788	.4793	.4798	.4803	.4808	.4812	.4817
2.1	.4821	.4826	.4830	.4834	.4838	.4842	.4846	.4850	.4854	.4857
2.2	.4861	.4864	.4868	.4871	.4875	.4878	.4881	.4884	.4887	.4890
2.3	.4893	.4896	.4898	.4901	.4904	.4906	.4909	.4911	.4913	.4916
2.4	.4918	.4920	.4922	.4925	.4927	.4929	.4931	.4932	.4934	.4936
2.5	.4938	.4940	.4941	.4943	.4945	.4946	.4948	.4949	.4951	.4952
2.6	.4953	.4955	.4956	.4957	.4959	.4960	.4961	.4962	.4963	.4964
2.7	.4965	.4966	.4967	.4968	.4969	.4970	.4971	.4972	.4973	.4974
2.8	.4974	.4975	.4976	.4977	.4977	.4978	.4979	.4979	.4980	.4981
2.9	.4981	.4982	.4982	.4983	.4984	.4984	.4985	.4985	.4986	.4986
3.0	.4987	.4987	.4987	.4988	.4988	.4989	.4989	.4989	.4990	.4990
3.1	.4990	.4991	.4991	.4991	.4992	.4992	.4992	.4992	.4993	.4993
3.2	.4993	.4993	.4994	.4994	.4994	.4994	.4994	.4995	.4995	.4995
3.3	.4995	.4995	.4995	.4996	.4996	.4996	.4996	.4996	.4996	.4997
3.4	.4997	.4997	.4997	.4997	.4997	.4997	.4997	.4997	.4997	.4998
3.5	.4998	.4998	.4998	.4998	.4998	.4998	.4998	.4998	.4998	.4998

Note : An entry in the table is the area under the standard normal curve between $x = 0$ and $x = z$ ($z \geq 0$). Areas for negative values of z can be obtained by symmetry.



$$A(z) = \int_0^z \frac{1}{\sqrt{2\pi}} e^{\frac{-x^2}{2}} dx$$