FUKIEN SECONDARY SCHOOL

S5 First Term Uniform Test (2021-2022)

Mathematics Extended Part Module 1

(1 hour 15 minutes)

Date: 12th November 2021 Name:_____

Time: 8:30 a.m.-9:45 a.m. Class: _____ No.: _____

Instructions to students:

1. The maximum score of this paper is 53.
2. Attempt ALL questions.
3. Write your answers in the spaces provided in this Question-Answer Book.
4. Unless otherwise specified, show your workings clearly.
5. Unless otherwise specified, numerical answers should be either exact or given to 4 decimal places.

1. (a) Expand e^{-3x} as a series in ascending powers of x as far as the term in x^3.

   (b) Hence, expand $\dfrac{(x+3)^4}{e^{3x}}$ as a series in ascending powers of x as far as the term in x^3.

(5 marks)

2. Consider the curve C : $y = x\sqrt{(2x-1)}$, where $x > \frac{1}{2}$.

(a) Find $\frac{dy}{dx}$.

(b) Using (a), find the equations of two tangents to the Curve C which are parallel to the straight line $2x - y = 0$.

(6 marks)

3 (a) Express $\frac{d}{dx}\left((x^6+1)\ln(x^2+1)\right)$ in the form $f(x)+g(x)\ln(x^2+1)$, where $f(x)$ and $g(x)$ are polynomials.

(b) Find $\int x^5\ln(x^2+1)dx$.

(7 marks)

4. A and B are two events. A' and B' are the complementary events of A and B respectively. Suppose $P(A)=\frac{1}{5}$, $P(A\cup B)=\frac{9}{20}$, $P(A|B)=\frac{1}{6}$ and $P(B)=k$, where $0<k<1$.

(a) Using $P(A|B)$, express $P(A\cap B)$ in terms of k.

(b) Find the value of k.

(c) Find $P(A'\cap B)$.

(d) Are the two events A' and B' mutually exclusive? Explain your answer.

(7 marks)

5. Let A and B be two exhaustive events of a certain sample space. Denote $P(B) = b$ and $P(A \cap B) = c$, where $0 < b < 1$ and $0 < c < 1$.

(a) Express $P(A)$ in terms of b and c.

(b) Suppose that $P(A|B) = \frac{1}{2}$ and $P(B|A) = \frac{2}{3}$.

(i) Find the values of b and c.

(ii) Are the events A and B independent? Explain your answer.

(7 marks)

6. In a factory, machines A and B produce 65% and 35% of all the products respectively. It is known that 5% of the products produced by machine A and 3% of the products produced by machine B are defective. Suppose a product is selected at random by an inspector.
   (a) Find the probability that the selected product is defective.
   (b) Find the probability that the selected product is produced by machine A, given that it is defective.
   (c) Find the probability that the selected product is produced by machine B, given that it is not defective.

(6 marks)

7. Let A and B be two events. It is given that $P(A|B) = \frac{3}{4}$, $P(B|A) = \frac{3}{8}$ and $P(A) = a$.

(a) Find $P(A \cap B)$ in terms of a.

(b) Find $P(B)$ in terms of a.

(c) It is given that $P(A' \cap B') = \frac{7}{16}$.

(i) Find the value of a.

(ii) Find the values of $P(A|B')$.

(7 marks)

8. Karen takes an English examination and a Chinese examination. The probability that Karen passes the English examination is 0.6. The probability that Karen passes at least one of the examinations is 0.92. Assume that her performances in the two examinations are independent of each other. Find the probability that Karen passes
   (a) the Chinese examination,
   (b) both examinations,
   (c) only one of the examinations,
   (d) none of the examinations.

(8 marks)

— End of Paper —