FUKIEN SECONDARY SCHOOL

S4 First Term Uniform Test (2021-2022)

Mathematics Compulsory Part

(1 hour 15 minutes)

Date: 9th November, 2021
Time: 8:30 a.m. - 9:45 a.m.

Name:____________________
Class: ________ No.: _______

Instructions to students:

1. This paper consists of TWO parts, Conventional Questions, Multiple-choice Questions and Bonus Question. There are Section A and Section B in Conventional Questions. Section A carries 49 marks, Section B carries 17 marks, Multiple-choice Questions carry 12 marks and Bonus Question carries 5 marks.

2. The maximum score of this paper is 78.

3. Attempt **ALL** questions in Conventional Questions and Multiple-choice Questions.
Write your answers in the spaces provided in this Question / Answer Book.

4. Unless otherwise specified, show your workings clearly.

5. Unless otherwise specified, numerical answers should either be exact or correct to 3 significant figures.

6. The diagrams in this paper are not necessarily drawn to scale.

Conventional Questions

Section A (49 marks)

1. Simplify $\dfrac{ab^7}{(a^3b^{-4})^{-2}}$ and express your answer with positive indices. (3 marks)

2. Let x, y and z be non-zero numbers such that $x : y = 3 : 4$ and $y = 5z$. Find $\frac{2x+3y}{2y-z}$. (3 marks)

3. Convert $1.\dot{3}\dot{6}$ into a fraction. (3 marks)

4. It is given that the quadratic equation $7x^2 + 6x + a = 2$ has real root(s). If a is a positive integer, how many possible values of a are there? Explain your answer. (3 marks)

5. Solve the following quadratic equations by any algebraic method.

(*Leave the radical sign '$\sqrt{\ }$' in the answers if necessary.*) (10 marks)

(a) $x^2 + 9x = 0$

(b) $49 - 4x^2 = 0$

(c) $3x^2 - x - 7 = 0$

(d) $(x - 3)(2x - 1) = -2$

6. It is given that $f(x) = x^2 + 2kx - 3$ and $f(-1) = 6$, where k is a constant.

(a) Find the value of k.

(b) Find the value of b such that $f(b) = -19$.

(4 marks)

7. Let $z = \dfrac{k+3i}{2-i}$, where k is a real number.

(a) Express z in the form $a + bi$.

(b) If the real part and the imaginary part of z are equal, find the value of k.

(5 marks)

8. It is given that α and β are two distinct real roots of $x^2+5x+2=0$, where $\alpha > \beta$.

(a) Find $\alpha\beta$ and $\alpha^2+\beta^2$.

(b) Form a quadratic equation in x with roots α^2 and β^2.

(6 marks)

9. Let $f(x) = x^3 + a^2x^2 + b^2x - 1$, where a and b are positive integers. It is given that $f(-1) = 1$.

   (a) By considering $(a + b)(a - b)$, or otherwise, write down the values of a and b.

   (b) Let $h(x) = f(x) - g(x)$, where $g(x) = x^3 + x^2 - 5x - 3$. Solve $h(x) = 0$.
   (Leave the radical sign '$\sqrt{\ }$' in the answers.)

(6 marks)

10. In Figure 1, the graph of $y = ax^2 + bx + c$ (where $a > 0$, $b \neq 0$ and $c \neq 0$) intersects the x-axis at $Q(c\ ,\ 0)$ only. The graph intersects the y-axis at P. Find the value of b.

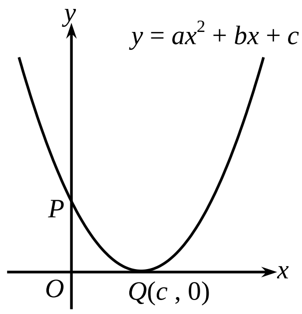


Figure 1

(6 marks)

Section B (17 marks)

In this section, answer EITHER part (X) OR part (Y) in each question. Do not answer both parts. If both parts in a question are attempted, only part (X) will be marked.

11. (X) Consider the graph of $y = 2x^2 - (2k + 6)x + 3k$.

(a) Show that the graph cuts the x-axis at two distinct points.

(b) Suppose the graph cuts the x-axis at P and Q. If the mid-point of PQ lies on a straight line L: $kx - y - 4k + 3 = 0$, find all the possible coordinates of the mid-point of PQ.

(8 marks)

(Y) Consider the graph of $y = 2x^2 - (2k + 6)x + 3k$.

(a) Show that the graph cuts the x-axis at two distinct points.

(b) Suppose the graph cuts the x-axis at P and Q. Find the coordinates of the mid-point of PQ in terms of k.

(4 marks)

12. (X) In a square paper card of side x cm, 4 identical squares of side 2 cm are cut from the corners. The remaining part is then folded to form a box without lid. Let $V(x)$ (in cm^3) be the capacity of the box.

(a) Express $V(x)$ in terms of x. (3 marks)

(b) Find the domain of $V(x)$. (2 marks)

(c) Steven claims that for each given capacity of the box, there is only one corresponding value of x. Do you agree? Explain your answer. (4 marks)

(Y) In a square paper card of side x cm, 4 identical squares of side 2 cm are cut from the corners. The remaining part is then folded to form a box without lid. Let $V(x)$ (in cm^3) be the capacity of the box.

(a) Express $V(x)$ in terms of x. (3 marks)

(b) If the capacity of the box is 18 cm^3, find the value of x. (2 marks)

Multiple-choice Questions (12 marks)

Write down the best answer to each question in the box.

1	2	3	4	5	6

1. Figure 2 shows the graph of $y = -x^2 + 3x - m + 1$. Find the range of values of m.

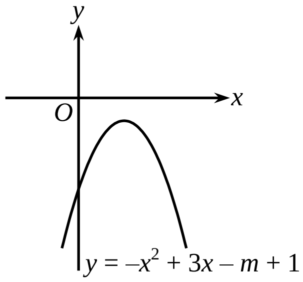


Figure 2

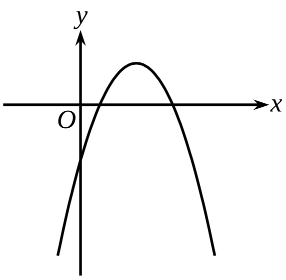
A. $m < 1$

B. $m > 1$

C. $m < \frac{13}{4}$

D. $m > \frac{13}{4}$

2. Which of the following can represent that y is a function of x?

I.


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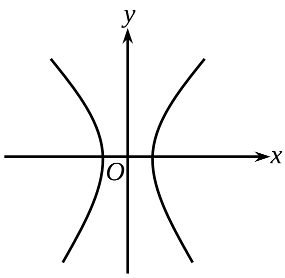

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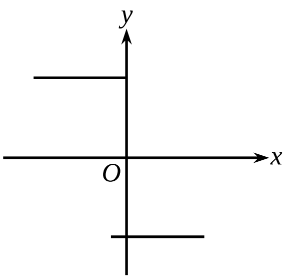
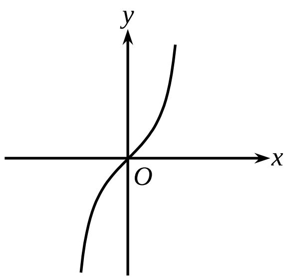


IV.


A. IV only

B. I and II only

C. I and IV only

D. I, III and IV only

3. Which of the following is the domain of the function $f(x) = \frac{x}{2x-1}$?

A. all real numbers

B. all real numbers except $\frac{1}{2}$

C. all real numbers except 1

D. all real numbers greater than $-\frac{1}{2}$

4. If $k > 2$, find the number of real roots of the equation $x^2 - 4x + 2k = 0$.
   A. 0
   B. 1
   C. 2
   D. 3

5. α and β are the roots of the equation $x^2 - kx + 2k - 1 = 0$. If $\alpha^3 + \beta^3 = k^3 - 3$, then $k =$

   A. $-\frac{1}{2}$.
   B. 1.
   C. $-\frac{1}{2}$ or 1.
   D. $\frac{1}{2}$ or -1.

6. If $\alpha \neq \beta$ and $\begin{cases} \alpha^2 - 7\alpha - 3 = 0 \\ \beta^2 - 7\beta - 3 = 0 \end{cases}$, then $\frac{1}{\beta} + \frac{1}{\alpha} =$

   A. $-\frac{7}{3}$.
   B. $-\frac{3}{7}$.
   C. $\frac{3}{7}$.
   D. $\frac{7}{3}$.

Bonus Question (5 marks)

1. Let $x = \frac{1}{2} - \frac{\sqrt{3}}{2}i$.

   (a) Simplify the following expressions.

   (i) $1 - x + x^2$

   (ii) x^3

   (b) Let k be a real number and $h = k - x + x^2 + x^3 - x^4 + x^5 + x^6 - x^7 + x^8$.

   (i) Show that h is a real number.

   (ii) If h does not exceed 4, find the largest value of k.

(5 marks)

END OF PAPER