FUKIEN SECONDARY SCHOOL
S4 First Term Uniform Test (2021-2022)
Mathematics Extended Part Module 1
(1 hour 15 minutes)

Date: 12th November 2021   Name:_____

Time: 8:30 a.m. - 9:45 a.m.   Class: _____ No.: _____

Instructions

1. Answer ALL questions in this paper. Write your answers in the Answer Book.
2. The maximum score of this paper is 67.
3. Unless otherwise specified, all workings must be clearly shown.
4. Unless otherwise specified, numerical answers should be either exact or given to 4 decimal places.
5. The diagrams in this paper are not necessarily drawn to scale.

1. Simplify $\dfrac{(2n)!(n+1)}{(2n+2)!}$.

(3 marks)

2. Simplify $C_{n-2}^{n+1} - C_{n-4}^{n-1}$, express the answer in terms of n .

(3 marks)

3. It is given that $\displaystyle\sum_{k=1}^{10} x_k = -8$, $\displaystyle\sum_{k=1}^{10} y_k = 32$ and $\displaystyle\sum_{k=1}^{10} x_k y_k = 24$. Find the value of

$\displaystyle\sum_{k=1}^{10} (2x_k - 1)(y_k + 1)$.

(3 marks)

4. It is given that $(1+px)^n = 1 - 27x + 36p^2 x^2 + $ terms involving higher powers of x , where p

    is a constant and n is a positive integer. Find the values of p and n .

(4 marks)

5. Find the constant term in the expansion of $\left(x^2 + \dfrac{1}{x}\right)^3 (x-3)^6$.

(5 marks)

6. Find the coefficients of the 5th term and the constant term in the expansion of $\left(\dfrac{1}{3x^2} - 2x\right)^9$ in

    descending powers of x.

(5 marks)

7.   (a)   Expand e^{-3x} as a series in ascending powers of x as far as the term in x^3.

   (b)   Hence, expand $\dfrac{(x+3)^4}{e^{3x}}$ as a series in ascending powers of x as far as the term in x^3.

(5 marks)

8.   Simplify the following expressions.   (6 marks)

   (a) $\ln(a^2-9)-\ln\sqrt{a+3}-\ln\sqrt{a-3}$

   (b) $\dfrac{\ln a^2 - \ln\sqrt[4]{a}}{\ln\sqrt{\dfrac{1}{a}}}$

   (c) $e^{3\ln 2 - 2\ln 3}$

9.   Solve the following equations.   (12 marks)

   (a) $\ln x = \dfrac{1}{3}(\ln 8 + 3\ln 2)$

   (b) $6e^x + e^{-x} = 5$

   (c) $3e^{2x} - 13e^x + 4 = 0$

   (d) $\sqrt{e^{x+1}} = 7^{2x-5}$

10.   (a)   Expand $(2e^x + 3)^3$ as a series in ascending powers of x as far as the terms in x^3.

   (b)   If the coefficient of x in the expansion of $\left(x+\dfrac{k}{x}\right)^2 (2e^x + 3)^3$ is -207, find the values of

   k.

(5 marks)

11. In a traffic accident, a toxic chemical is discharged into a lake. The concentration $C\,g/m^3$ of the chemical in the lake t days after the accident is given by $C = Ae^{-kt}$, where A and k are constants. When $t = 1$, $C = 14.46$. When $t = 2$, $C = 9.50$.

    (a) Find the values of A and k, correct to 2 significant figures.

    (b) After the accident, the lake is closed for safety. If the concentration of the chemical drops below $0.3g/m^3$, the lake will open again. Estimate for how long the lake will be closed after the accident. (Give your answer correct to the nearest day.)

                                                                           (6 marks)

12. In an experiment, the temperature N (in °C) of a certain liquid can be modelled by $N = \dfrac{15e^{bt}}{e^{bt} + a}$, where a and b are constants, and $t(\geq 0)$ is the number of minutes elapsed since the start of the experiment.

    (a) Prove that $N = \dfrac{15}{ae^{-bt} + 1}$.

    (b) Express $\ln\left(\dfrac{15}{N} - 1\right)$ as a linear function of t.

    (c) It is given that the slope and the intercept on the horizontal axis of the graph of the linear function obtained in (b) are -0.2 and $5\ln 74$ respectively.

        (i) Find a and b.

        (ii) Find the temperature of the liquid at the start of the experiment.

        (iii) At least after how many minutes from the start of the experiment will the temperature of the liquid be 10°C or above? (Give your answer correct to the nearest integer.)

                                                                        (10 marks)

- End of Paper -