FUKIEN SECONDARY SCHOOL

S5 Final Examination (2020-2021)

Mathematics Compulsory Part Paper 1

(2 hours 15 minutes)

Date: 11th June 2021 Name: _____

Time: 8:30 a.m. - 10:45 a.m. Class: _____ No. :_____

Instructions to students:

1. This paper consists of THREE parts, Section A(1), Section A(2) and Section B.
2. The maximum score of this paper is 105.
3. Attempt ALL questions. Write your answers in the spaces provided in this Question / Answer Book.
4. Unless otherwise specified, show all workings clearly.
5. Unless otherwise specified, numerical answers should either be exact or correct to 3 significant figures.
6. The diagrams in this paper are not necessarily drawn to scale.

Section A(1) (35 marks)

1. Simplify $\dfrac{(x^{-2}y^3)^3}{(x^{-4}y^2)^{-2}}$ and express your answer with positive indices. (3 marks)

2. Factorize
   (a) $x^2 - 5xy + 6y^2$,
   (b) $x^2 - 5xy + 6y^2 + 13x - 39y$. (3 marks)

3. Consider the formula $\frac{3a-2}{a+2b}=2$.

(a) Make b the subject of the above formula.

(b) If the value of a is decreased by 8, write down the change in the value of b.

(3 marks)

4. The total number of books owned by Amy and Billy is 84. If Amy buys 12 books from a book store, the number of books owned by her will be 3 times that owned by Billy. Find the number of books owned by Billy. (4 marks)

5. The cost of a dictionary is \$50. If it is sold at a discount of 10% on its marked price, then the percentage profit is 44%. Find the marked price of the dictionary.

(3 marks)

6. (a) Solve the inequality $\frac{5x+46}{3} \leq 2(3x-1)$.

(b) Find all integers satisfying both inequalities $\frac{5x+46}{3} \leq 2(3x-1)$ and $32 - 4x > 0$.

(4 marks)

7. A piece of wire 100 cm long is bent to form a rectangle with length x cm.
   (a) If the area of the rectangle is A cm^2, express A in terms of x.
   (b) Fanny claims that the area of the rectangle can be greater than 650 cm^2. Do you agree? Explain your answer.

(5 marks)

8. The 6th term and the 14th term of an arithmetic sequence are 27 and –5 respectively. If the sum of the first m terms of the sequence is –330, find m.

(5 marks)

9. The coordinates of the points A and B are $(3, -6)$ and $(-2, 5)$ respectively. A' is the reflection image of A with respect to the y-axis. B is rotated anticlockwise about the origin O through $270°$ to B'.

   (a) Write down the coordinates of A' and B'.

   (b) Let P be a moving point in the rectangular coordinate plane such that it maintains a fixed distance from A'. If the locus of P passes through B', find its equation.

(5 marks)

Section A(2) (35 marks)

10. The production cost of a rectangular container of volume V cm^3 is \$$C$. It is given that C is partly constant and partly varies as $\sqrt{V}$. When $V = 2500$, $C = 31$ and when $V = 3600$, $C = 36$.
    (a) Find the production cost of a container A of volume 2025 cm^3. (4 marks)
    (b) A container B is similar to the container A described in (a). If the surface area of the container B is 2.56 times that of the container A, find the production cost of the container B correct to the nearest dollar. (3 marks)

11. Let $f(x) = x(x-4)^2 + ax + b$, where a and b are constants. It is given that $x - 3$ is a factor of $f(x)$. When $f(x)$ is divided by $x - 1$, the remainder is 2.

(a) Find a and b. (3 marks)

(b) Someone claims that the equation $f(x) = 0$ has at least one irrational root. Do you agree? Explain your answer. (3 marks)

12. In Figure 1, $PQRS$ is a circle. PR and QS intersect at T, and $\angle STR = 72°$. It is given that $PR = PS$ and $\overset{\frown}{PQ} = \frac{1}{2}\overset{\frown}{RS}$.

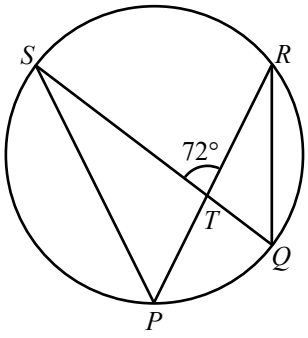

Figure 1

(a) Is QS a diameter of the circle? Explain your answer. (4 marks)

(b) If $\overset{\frown}{RS} = 8$, find the circumference of the circle. (2 marks)

13. CP is the median of AB in $\triangle ABC$, where A is a point lying on the y-axis. It is given that the coordinates of C are $(c, 2)$, and the equations of AB and CP are $x + y - 6 = 0$ and $5x - 3y + 66 = 0$ respectively.

(a) Find the coordinates of A and B. (3 marks)

(b) Find the value of c. Hence find the equation of BC. (2 marks)

(c) Q is a point lying on AC such that the area of $\triangle PAQ$ is 36. Let $AQ : QC = r : 1$. Find the value of r. (3 marks)

14. An inverted right circular conical vessel of base radius 96 cm and height 180 cm is held vertically and it contains some water with volume 8640π cm^3.

(a) Find the depth of water in the vessel. (3 marks)

(b) It is known that 195 hemispherical containers of radius 12 cm are fully filled with water. The water in the containers is now poured into the vessel without overflowing. Someone claims that the increase in the wet curved surface of the vessel is at least 30 000 cm^2. Do you agree? Explain your answer. (5 marks)

Section B (35 marks)

15 In Figure 2, M_1, M_2, M_3, …are similar figures. The perimeter and the area of M_1 are 16 cm and 8 cm^2 respectively. The perimeter of each succeeding figure is $\frac{5}{4}$ of that of the previous one.

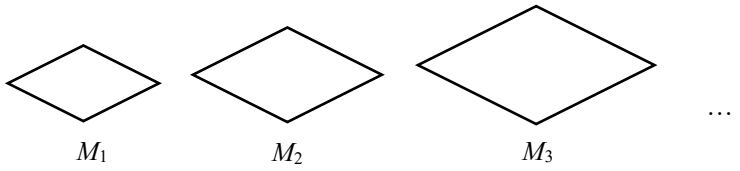

Figure 2

(a) (i) Find the perimeter of M_{25} correct to the nearest cm.

(ii) Find the sum of the perimeters of the first 25 figures correct to the nearest cm.

(3 marks)

(b) Find the sum of the areas of M_1 to M_{25} correct to the nearest cm^2. (2 marks)

16. Let $f(x) = -3(x - 2)^2 + 5$. The graph of $y = g(x)$ is obtained by reflecting the graph of $y = f(x)$ with respect to the x-axis, followed by translating leftwards by k units, where $k > 0$. It is known that the y-intercept of the graph of $y = g(x)$ is 7.

(a) Find k. (3 marks)

(b) The graph of $y = h(x)$ is obtained by translating the graph of $y = g(x)$ rightwards by 5 units, and then upwards by 10 units. Does the graph of $y = h(x)$ intersect the graph of $y = f(x)$? Explain your answer. (2 marks)

17.(a) In Figure 3, the equation of the straight line L_1 is $2x + 3y = 270$ and the x-intercept of the straight line L_2 is 120. L_1 and L_2 intersect at the point (60, 50). The shaded region (including the boundary) represents the solution of a system of inequalities. Write down the system of inequalities. (2 marks)

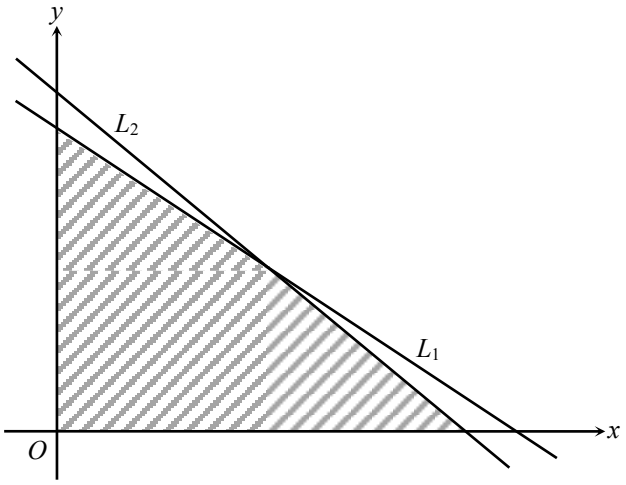

Figure 3

(b) A factory produces two types of cupboards, *X* and *Y*. Each cupboard *X* requires 5 man-hours for assembly and 2 man-hours for packing while each cupboard *Y* requires 6 man-hours for assembly and 3 man-hours for packing. In a certain month, the factory has 600 man-hours available for assembly and 270 man-hours available for packing. The profits for producing a cupboard *X* and a cupboard *Y* are \$540 and \$650 respectively. Suppose x cupboards of type *X* and y cupboards of type *Y* are sold that month.

(i) Write down all the constraints on x and y.

(ii) Find the maximum profit that can be made by selling all the cupboards produced that month. (3 marks)

18. Figure 4(a) shows a solid pyramid $VPQRS$ with a rectangular base, where $PQ = 21$ cm, $PS = 12$ cm, $VP = VS = 30$ cm and $\angle VQP = \angle VRS = 110°$.

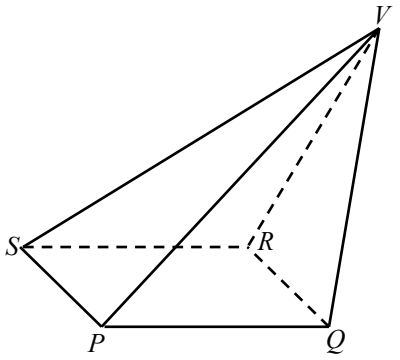

Figure 4(a)

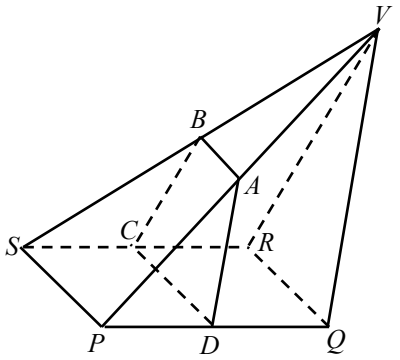

Figure 4(b)

(a) Find $\angle VPQ$. (2 marks)

(b) A, B, C and D are the mid-points of VP, VS, RS and PQ respectively. A geometric model is made by cutting off $CSPDAB$ from $VPQRS$ as shown in Figure 4(b).

(i) Find the area of the trapezium $ABCD$.

(ii) Someone claims that among the trapeziums $ABCD$, $ABSP$ and $ABRQ$, the trapezium $ABCD$ has the least area. Do you agree? Explain your answer.

(7 marks)

19. $\triangle ABC$ is an isosceles triangle, where $AB = BC$. It is given that the coordinates of A and C are $(-3, -1)$ and $(7, -1)$ respectively, the y-coordinate of B is positive, and the area of $\triangle ABC$ is 25.
    (a) (i) Find the coordinates of B. Hence, find the equation of the perpendicular bisector of AB.
    (ii) Find the coordinates of the circumcentre of $\triangle ABC$. (7 marks)
    (b) Let S be the circle which passes through A, B and C.
    (i) Find the equation of S.
    (ii) Someone claims that the area of S is less than 4 times the area of the inscribed circle of $\triangle ABC$. Do you agree? Explain your answer. (4 marks)

END OF PAPER