FUKIEN SECONDARY SCHOOL
S5 Final Examination (2020-2021)
Mathematics Extended Part Module 1
(2 hours 30 minutes)

Date: 8th June 2021 Name:____________________
Time: 8:30 a.m. - 11:00 a.m. Class: ________ No.: _______

Instructions to students:

1. This paper consists of Section A (50 marks) and Section B(50 marks).

2. Attempt ALL questions. Write your answers in the Answer Book.

3. Unless otherwise specified, show your workings clearly.

4. Unless otherwise specified, numerical answers should be either exact or given to 4 decimal places.

5. The diagrams in this paper are not necessarily drawn to scale.

Section A (50 marks)

1. Let A and B be two events. Suppose that $P(B) = 0.325$, $P(B \mid A) = 0.25$ and $P(B' \mid A') = 0.593\ 75$, where A' and B' are complementary events of A and B respectively.
   (a) Find $P(A)$.
   (b) Find $P(A \mid B')$.
   (c) Find $P(A' \cup B)$.

(5 marks)

2. Molly is suggested to take a vitamin capsule every morning. If Molly gets up late in a morning, the probability for her to forget to take a vitamin capsule in that morning is 0.35; otherwise, the probability for her to forget to take a vitamin capsule in that morning is 0.25. It is known that the probability for Molly to get up late in a morning is 0.2.
   (a) Find the probability that Molly forgets to take a vitamin capsule in a certain morning.
   (b) Given that Molly forgets to take a vitamin capsule in a certain morning, find the probability that she gets up late in that morning.
   (c) Find the probability that Molly forgets to take vitamin capsules in at least 5 mornings in a certain week.

(6 marks)

3. The following table shows the probability distribution of a discrete random variable X, where m and p are positive constants.

x	-1	4	6	8
$P(X = x)$	$2p$	$0.2 - 3p$	$0.6 - 3p$	$mp + 0.2$

   (a) Find m.
   (b) If $10\text{Var}(X + m) = \text{E}(m^2 X - 29)$, find p.

(7 marks)

4. The number of purchase orders for a product received by a company in a week follows a distribution with a mean of μ and a standard deviation of 3.5. The company randomly collects a sample of 49 weeks and an approximate λ% confidence interval for μ is found to be (10.98, 13.22).
   (a) Find the sample mean of the number of purchase orders for the product in a week.
   (b) Find λ.
   (c) Find the least number of weeks required for sampling such that the probability for the sample mean of the number of purchase orders for the product in a week to differ from μ by more than 0.5 is at most 0.038.

(7 marks)

5. (a) Expand $e^{3x}+e^{-2x}$ in ascending powers of x as far as the term in x^2.

(b) Find the coefficient of x^2 in the expansion of $(e^{3x}+e^{2x}-1)\left(4-\dfrac{x}{2}\right)^6$.

(5 marks)

6. Let $f(x)$ be a continuous function such that $f'(x)=\dfrac{e^{-x}-me^{-2x}}{e^{-2x}-e^{-x}+3}$ for all real numbers x. It is given that $f(x)$ attains its minimum value at $x=\ln 2$.

(a) Find m.

(b) It is given that the extreme value of $f(x)$ is $\ln 11$.

(i) Find $f(x)$.

(ii) Find $\lim_{x\to\infty} f(x)$.

(6 marks)

7. Consider the curve $C: y=\dfrac{kx^2}{\sqrt{x^2+6}}$, where $k \neq 0$.

(a) Find $\dfrac{dy}{dx}$.

(b) Let L_1 and L_2 be the two tangents to C at $x=\sqrt{3}$ and $x=-\sqrt{3}$ respectively. If the y-intercept of L_1 is $-\dfrac{2\sqrt{3}}{3}$, find the equation of L_2.

(7 marks)

8. Let $f(x)=1-\dfrac{5(2^{-x})}{2^{-x}+1}$.

(a) Using integration by substitution, find $\int f(x)dx$.

(b) Let C be the curve of $y=f(x)$.

(i) Find the x-intercept of C.

(ii) Find the exact value of the area of the region bounded by C, the x-axis and the y-axis.

(7 marks)

Section B (50 marks)

9. In a city, the hourly wages of cleaners who provide one-off cleaning services before Chinese New Year follow a normal distribution with a mean of $\$\mu$ and a standard deviation of \$10.

(a) A survey is conducted to estimate μ.

(i) A sample of 16 cleaners is randomly selected, and the stem-and-leaf diagram below shows the distribution of their hourly wages (in \$):

Stem (tens)	Leaf (units)
14	0 2 3 5 5 9
15	0 0 1 7 8
16	0 0 5
17	0 5

Find a 95% confidence interval for μ.

(ii) Find the least sample size to be taken such that the width of a 99.5% confidence interval for μ is at most 5.

(7 marks)

(b) Suppose that $\mu = 154$. The hourly wages of \$145 or below, between \$145 and \$170, and \$170 or above of a cleaner for providing one-off cleaning services before Chinese New Year are regarded as low, medium and high respectively.

(i) Find the probability that the hourly wage of a randomly selected cleaner for providing such cleaning services is regarded as high.

(ii) A sample of 15 cleaners for providing such cleaning services are randomly selected. Given that more than 10 cleaners with medium hourly wages are selected in this sample, find the probability that at least 3 cleaners with high hourly wages are selected.

(6 marks)

10. In a smartphone shop, the sales volume of a model of smartphone made by a salesperson in a week will be considered as high if more than 2 smartphones of this model are sold by the salesperson in that week. To encourage the sales volume of smartphone P, the shop owner has implemented a reward scheme. A salesperson will receive different amounts of reward according to the number of weeks with high sales volume of smartphone P made by the salesperson in four weeks as shown below.

Number of weeks with high sales volume of smarphone P in four weeks	0	1	2	3	4
Reward	\$0	\$120	\$300	\$750	\$1 200

James is a salesman of the shop, and past records show that the number of smartphones P sold by him in one week follows a Poisson distribution with a variance of 2.6.

(a) Find the probability that the sales volume of smartphone P made by James is high in a certain week. (2 marks)

(b) Find the expected amount of the reward received by James for the sales volume of smartphone P made by him in four weeks. (3 marks)

(c) The shop owner suggests a new reward scheme. Under this new reward scheme, a salesperson will be rewarded with \$400 if more than 5 smartphones P are sold by the salesperson in four weeks. Otherwise, the salesperson will not be rewarded. James claims that this new reward scheme is more favourable to him. Do you agree? Explain your answer. (2 marks)

(d) The number of smartphone Q sold by James in a week follows a Poisson distribution with a mean of 3. It is assumed that the weekly numbers of smartphones P and Q sold by James are independent.

(i) Find the probability that the sales volumes of both smartphones P and Q made by James are high in a certain week.

(ii) Given that the sales volumes of both smartphones P and Q made by James are high in a certain week, find the probability that James sells more than 7 smartphones P and Q in total in that week.

(5 marks)

11. In a factory, the rate of change of the number of light bulbs A produced (in thousand per hour) can be modelled by

$$F(t) = t\ln(\sqrt{t}+10),$$

where $t(0 \le t \le 8)$ is the number of hours elapsed since the start of the production of light bulbs A on a particular day. Denote the number of light bulbs A produced by the factory in the first 2 hours after the start of the production of light bulbs A on that day by α thousand. Let α_1 be the estimate of α by using the trapezoid rule with 5 sub-intervals.

(a) (i) Find α_1.

(ii) Is α_1 an over-estimate or an under-estimate? Explain your answer.

(6 marks)

(b) The factory also produces light bulbs B. The rate of change of the number of light bulbs B produced (in thousand per hour) can be modelled by

$$G(t) = \frac{t^2+t+4}{t+2},$$

where $t(0 \le t \le 8)$ is the number of hours elapsed since the start of the production of light bulbs B on a particular day.

(i) Find the number of light bulbs B produced by the factory in the first 2 hours after the start of the production of light bulbs B on that day.

(ii) The manager of the factory claims that in the first 2 hours after the start of the production of light bulbs, the difference between the numbers of light bulbs A and light bulbs B produced exceeds 16% of the number of light bulbs B produced. Do you agree? Explain your answer.

(6 marks)

12. In an experiment, the temperature x (in °C) of a certain liquid can be modelled by

$$x = 150 - \frac{1\ 040}{5(\lambda^{kt}) + 8},$$

where $t (\geq 0)$ is the number of minutes elapsed since the start of the experiment, λ and k are constants, and $\lambda > 1$.

(a) (i) Express $(x-20)(x-150)$ in terms of λ, k and t.

(ii) The laboratory technician claims that the temperature of the liquid lies between 20°C and 150°C. Do you agree? Explain your answer.

(4 marks)

(b) It is given that $\frac{dx}{dt} = \frac{\ln \lambda}{3\ 250}(x-20)(x-150)$.

(i) Find k.

(ii) Describe how x and $\frac{dx}{dt}$ varies during the first 30 minutes after the start of the experiment.

(iii) If the change in the temperature of the liquid during the first 25 minutes after the start of the experiment is $\frac{2\ 400}{61}$°C, find λ.

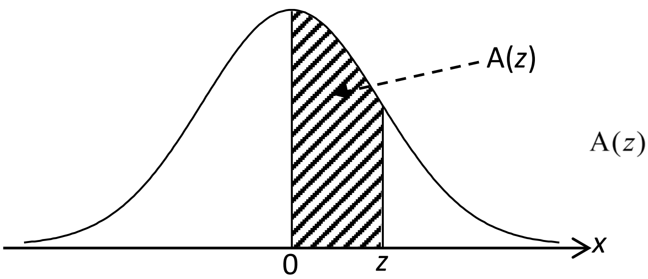
(9 marks)

End of Paper

Standard Normal Distribution Table

z	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
0.0	.0000	.0040	.0080	.0120	.0160	.0199	.0239	.0279	.0319	.0359
0.1	.0398	.0438	.0478	.0517	.0557	.0596	.0636	.0675	.0714	.0753
0.2	.0793	.0832	.0871	.0910	.0948	.0987	.1026	.1064	.1103	.1141
0.3	.1179	.1217	.1255	.1293	.1331	.1368	.1406	.1443	.1480	.1517
0.4	.1554	.1591	.1628	.1664	.1700	.1736	.1772	.1808	.1844	.1879
0.5	.1915	.1950	.1985	.2019	.2054	.2088	.2123	.2157	.2190	.2224
0.6	.2257	.2291	.2324	.2357	.2389	.2422	.2454	.2486	.2517	.2549
0.7	.2580	.2611	.2642	.2673	.2704	.2734	.2764	.2794	.2823	.2852
0.8	.2881	.2910	.2939	.2967	.2995	.3023	.3051	.3078	.3106	.3133
0.9	.3159	.3186	.3212	.3238	.3264	.3289	.3315	.3340	.3365	.3389
1.0	.3413	.3438	.3461	.3485	.3508	.3531	.3554	.3577	.3599	.3621
1.1	.3643	.3665	.3686	.3708	.3729	.3749	.3770	.3790	.3810	.3830
1.2	.3849	.3869	.3888	.3907	.3925	.3944	.3962	.3980	.3997	.4015
1.3	.4032	.4049	.4066	.4082	.4099	.4115	.4131	.4147	.4162	.4177
1.4	.4192	.4207	.4222	.4236	.4251	.4265	.4279	.4292	.4306	.4319
1.5	.4332	.4345	.4357	.4370	.4382	.4394	.4406	.4418	.4429	.4441
1.6	.4452	.4463	.4474	.4484	.4495	.4505	.4515	.4525	.4535	.4545
1.7	.4554	.4564	.4573	.4582	.4591	.4599	.4608	.4616	.4625	.4633
1.8	.4641	.4649	.4656	.4664	.4671	.4678	.4686	.4693	.4699	.4706
1.9	.4713	.4719	.4726	.4732	.4738	.4744	.4750	.4756	.4761	.4767
2.0	.4772	.4778	.4783	.4788	.4793	.4798	.4803	.4808	.4812	.4817
2.1	.4821	.4826	.4830	.4834	.4838	.4842	.4846	.4850	.4854	.4857
2.2	.4861	.4864	.4868	.4871	.4875	.4878	.4881	.4884	.4887	.4890
2.3	.4893	.4896	.4898	.4901	.4904	.4906	.4909	.4911	.4913	.4916
2.4	.4918	.4920	.4922	.4925	.4927	.4929	.4931	.4932	.4934	.4936
2.5	.4938	.4940	.4941	.4943	.4945	.4946	.4948	.4949	.4951	.4952
2.6	.4953	.4955	.4956	.4957	.4959	.4960	.4961	.4962	.4963	.4964
2.7	.4965	.4966	.4967	.4968	.4969	.4970	.4971	.4972	.4973	.4974
2.8	.4974	.4975	.4976	.4977	.4977	.4978	.4979	.4979	.4980	.4981
2.9	.4981	.4982	.4982	.4983	.4984	.4984	.4985	.4985	.4986	.4986
3.0	.4987	.4987	.4987	.4988	.4988	.4989	.4989	.4989	.4990	.4990
3.1	.4990	.4991	.4991	.4991	.4992	.4992	.4992	.4992	.4993	.4993
3.2	.4993	.4993	.4994	.4994	.4994	.4994	.4994	.4995	.4995	.4995
3.3	.4995	.4995	.4995	.4996	.4996	.4996	.4996	.4996	.4996	.4997
3.4	.4997	.4997	.4997	.4997	.4997	.4997	.4997	.4997	.4997	.4998
3.5	.4998	.4998	.4998	.4998	.4998	.4998	.4998	.4998	.4998	.4998

Note : An entry in the table is the area under the standard normal curve between $x = 0$ and $x = z$ ($z \geq 0$). Areas for negative values of z can be obtained by symmetry.



$$\mathrm{A}(z) = \int_0^z \frac{1}{\sqrt{2\pi}} e^{\frac{-x^2}{2}}\, dx$$