Fukien Secondary School

S4 Final Examination (2020-2021)

Mathematics Compulsory Part Paper I

(2 hours)

Date: 16th June, 2021 Name:_____

Time: 8:30 a.m.- 10:30 a.m. Class: _____ No. : _____

Instructions to students:

1. This paper consists of THREE parts, Section A(1), Section A(2) and Section B. Section A(1) carries 40 marks. Section A(2) carries 35 marks. Section B carries 25 marks.

2. The maximum score of this paper is 100.

3. Attempt ALL questions and write your answers in the spaces provided in this Question / Answer Book.

4. Unless otherwise specified, show your workings clearly.

5. Unless otherwise specified, numerical answers should either be exact or correct to 3 significant figures.

6. The diagrams in this paper are not necessarily drawn to scale.

Section A(1) (40 marks)

1. Simplify $\dfrac{\left(xy^2\right)^2}{x^{-7}y^6}$ and express your answer with positive indices.

(3 marks)

2. (a) Round off 202.1495 to 2 significant figures.
   (b) Round down 202.1495 to 2 decimal places.
   (c) Round up 202.1495 to the nearest thousand.

(3 marks)

3. Factorize
   (a) $4a - 4 - a^2$
   (b) $2b - ab + 4a - 4 - a^2$

(3 marks)

4. Make m the subject of the formula $\frac{p+4m}{3-p}=2m$.

(3 marks)

5. In a polar coordinate system, the polar coordinates of the points *P* and *Q* are $(5,52°)$ and $(10,112°)$.

(a) Let O be the pole. Someone claims that OP is perpendicular PQ. Do you agree? Explain your answer.

(b) Find PQ .

(4 marks)

6. A sum of money \$424 is first divided into two parts. Albert and Bob then further divide the two parts between themselves, in the ratio 7:2 for the first part and 1:2 for the second part. If they receive equal amounts in total, find the amount of the second part.

(4 marks)

7. If the graph of $y = 4x^2 - 12x + (k - 1)$ cuts the x-axis at two points, find

(a) the range of values of k,

(b) the x-intercepts of the graph when k is the largest integer in (a).

(4 marks)

8. It is given that $f(x)=9(2^x)$, $g(x)=3\left(\frac{1}{2}\right)^x$ and $h(x)=\frac{f(x)}{g(x)}$.

(a) Express $h(x)$ in the form ka^x.

(b) Find the value of b such that $h(b)=195$.

(4 marks)

9. Let $z=(3+i)+k\left(\frac{1}{3+i}\right)$, where k is a real number. If the imaginary part of z is 2, find

(a) the value of k,

(b) z.

(4 marks)

10. Let $f(x) = 2x^2 - 13x + k$, where k is a constant. If $f(x)$ is divisible by $x - 5$, find the remainder when $f(x)$ is divided by $2x + 5$.

(4 marks)

11. (a) Simplify $\dfrac{\cos(90° + x)\sin(270° + x)}{\sin(180° - x)}$.

(b) Hence, solve $\dfrac{\cos(90° + x)\sin(270° + x)}{\sin(180° - x)} = \dfrac{2}{3}$, for $0° \le x \le 360°$.

(Give your answers correct to 1 decimal place.)

(4 marks)

Section A(2) (35 marks)

12. (a) Let $f(x) = -x^2 + 24x$. Using the method of completing the square, find the coordinates of the vertex of the graph $y = f(x)$.

(2 marks)

(b) The length of a fence is 24m. Darren cuts the fence into two parts. One part is used to enclose his small farmland of area $A\,m^2$, which is in the shape of an equilateral triangle. The other part of length d m is used to divide the farmland into two equal parts as shown in Figure 1.

(i) Express A in terms of d.

(ii) Darren claims that the area of his farmland cannot be greater than $25m^2$. Do you agree? Explain your answer.

(4 marks)

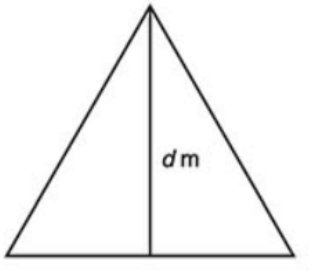

Figure 1

13. Let $f(x)=(x+m)(x+n)(3x-1)+2$, where m and n are positive integers and $m>n$. When $f(x)$ is divided by $x-1$, the remainder is 14.

(a) (i) Show that $(m+1)(n+1)=6$.

(ii) Write down the values of m and n.

(4 marks)

(b) Let $g(x)=k(x^2+x+2)$, where k is a non-zero constant. It is given that $f(x)-g(x)$ is divisible by $x+1$.

(i) Find the value of k.

(ii) Edmond claims that only one of the roots of the equation $f(x)-g(x)=0$ is real. Do you agree ? Explain your answer.

(5 marks)

14. (a) It is given that α and β are the roots of the equation $x^2 + x - 5 = 0$. Find the value of $\frac{\alpha + \beta}{2}$.

(1 mark)

(b) On a rectangle coordinate plane, the straight line $y = mx + 3$ intersects the quadratic curve $y = 2x^2 - 2x - 7$ at two points *A* and *B*. It is given that the *x*-coordinate of the mid-point of *AB* is $-\frac{1}{2}$.

(i) Find the value of m.

(ii) Hence, find the *y*-coordinate of the mid-point of *AB*.

(5 marks)

15. Figure 2(a) shows a piece of equilateral triangular paper *ABC* of side 66cm. *D* and *E* are points on *BC* such that $BD = 11cm$ and $EC = x\,cm$. The paper *ABC* is folded along *AD* and *AE*, so that the edge *AC* coincides with the edge *AB* to form a quadrilateral *ADFE* as shown in Figure 2(b). (i.e. *A, D, E* and *F* lie on the same horizontal ground.)

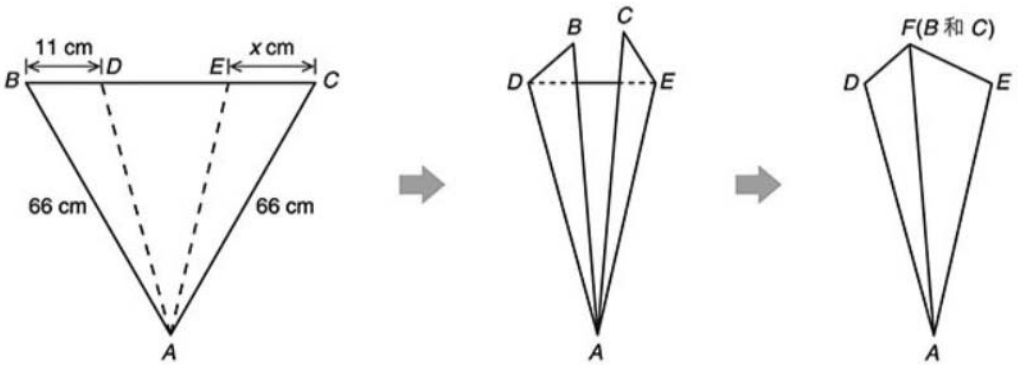

Figure2(a) Figure 2(b)

Refer to Figure 2(b).

(a) (i) Find $\angle DFE$.

(ii) Hence, find the value of x.

(4 marks)

(b) Find the area of quadrilateral *ADFE*.

(3 marks)

16. In Figure 3, *ABCD* is a square and *BDGE* is a parallelogram. *BCF*, *DCE* and *EFG* are straight lines.

(a) Prove that

(i) $\Delta BCD \sim \Delta FCE$,

(ii) $\Delta BCE \cong \Delta DCF$.

(4 marks)

(b) It is given that $\angle CBE = 30°$. For the sides *EF* and *FG*, which one is longer? Explain your answer.

(3 marks)

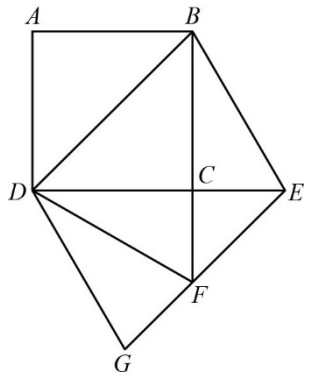

Figure 3

Section B (12-25 marks)

In this section, answer part (X) or part (Y) in each question. Do not answer both parts. If both parts in a question are attempted, only part (X) will be marked.

17X. Consider the equation $\dfrac{\log_2 a \cdot \log_2 x}{\log_2\left(\dfrac{x}{2}\right)} - \dfrac{\log_2 b}{\log_2 x^2} = 0$, where $a > 0$, $b > 0$ and $a, b \neq 1$.

(a) Set up a quadratic equation in $\log_2 x$. (1 mark)

(b) If the equation has a double real root, express b in terms of a. (3 marks)

(c) Solve $\dfrac{\log_2 x}{2\log_2\left(\dfrac{x}{2}\right)} - \dfrac{4}{\log_2 x^2} = 0$. (2 marks)

17Y. Solve $\left[\log_3\left(x - \dfrac{1}{3}\right)\right]^2 + 4\log_3\left(x - \dfrac{1}{3}\right) + 3 = 0$.

(3 marks)

18X. Consider the function $f(x) = A(x-7)(x+2) + B(x+1) + 4$, where A and B are constants. It is given that $f(7) = 12$ and $f(-1) = -4$.

(a) Find the values of A and B. (2 marks)

(b) The figure below shows three sketches of the graph of $y = f(x)$ drawn by three students. The teacher points out that the three sketches are all incorrect. Explain why each of the sketches is incorrect.

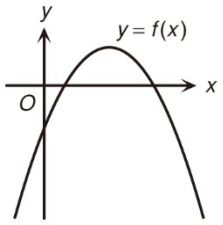

Figure 4(1)

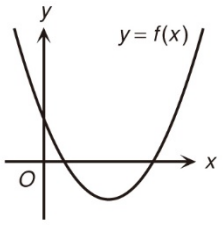

Figure 4(2)

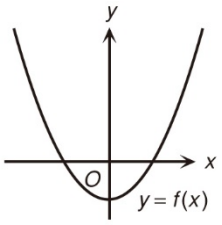

Figure 4(3)

(6 marks)

(c) Let α and β be the two x-intercepts of the graph of $y = f(x)$. If the two x-intercepts of the graph of $y = x^2 + bx + c$ are $\alpha - 2\beta$ and $\beta - 2\alpha$, find the values of b and c. (3 marks)

18Y. The Figure 5 shows the graph of $y = 2x^2 - 3x - 9$. It cuts the y-axis at P and Q is its vertex. Find the area of $\triangle OPQ$.

(5 marks)

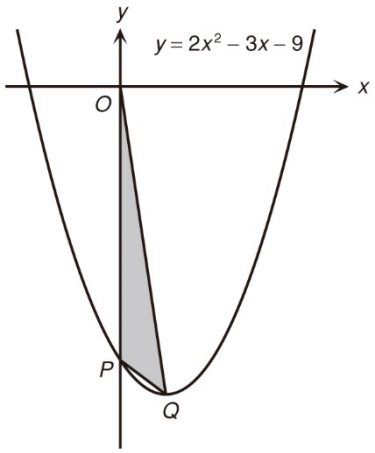

Figure 5

19X. Figure 6 shows a geometric model *ABCD* in the shape of a tetrahedron. It is given that $AD = 15$ cm, $BC = 17$ cm, $CD = 27$ cm, $\angle ABD = 58°$, $\angle ADB = 65°$ and $\angle ABC = 116°$.

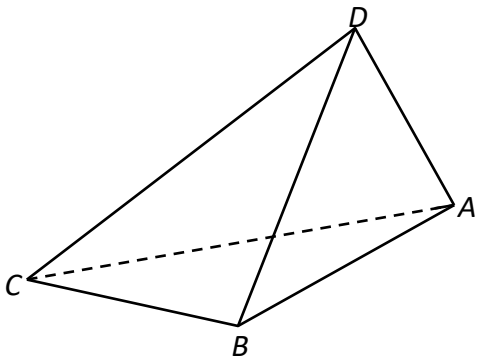

Figure 6

(a) Find *AB* and *AC*.

(4 marks)

(b) Let *K* be a point on *AD* such that $BK \perp AD$. Someone claims that $\angle BKC$ is the angle between the face *ABD* and the face *ACD*. Do you agree? Explain your answer.

(4 marks)

19Y. In Figure 7, *ABCD* is a tetrahedron, where $\Delta ABC \cong \Delta ADC$. It is given that $BD = 9$ cm, $BC = DC = 8$ cm and $\angle ACB = \angle ACD = 50°$. Find the angle between the planes *ABC* and *ACD*.

(4 marks)

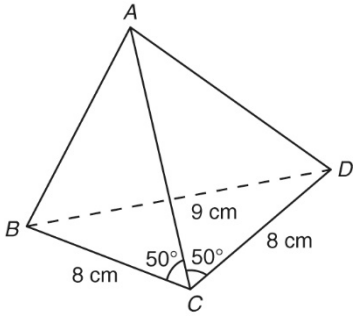

Figure 7

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