FUKIEN SECONDARY SCHOOL
S4 Final Examination (2020-2021)
Mathematics Extended Part Module 1
(2 hours)

Date: 22nd June 2021 Name:____________________

Time: 8:30 a.m. – 10:30 a.m. Class: ________ No.: _______

Instructions

1. This paper consists of Section A (60 marks) and Section B(40 marks). Answer ALL questions in this paper.
2. Write your answers in the Answer Book.
3. Unless otherwise specified, show your workings clearly .
4. Unless otherwise specified, numerical answers should be either exact or given to 4 decimal places.
5. The diagrams in this paper are not necessarily drawn to scale.

Section A (60 marks)

1. (a) Expand e^{-6x} in ascending powers of x as far as the term in x^2.

   (b) Find the coefficient of x^2 in the expansion of $\frac{(2+x)^5}{e^{6x}}$.

(5 marks)

2. Solve $6C_2^{n+1} + C_2^n = 534$.

(5 marks)

3. It is given that $y = e^{kx}$ where k is a constant.

   (a) Find $\frac{dy}{dx}$ and $\frac{d^2y}{dx^2}$.

   (b) Hence, find the values of k if $y = e^{kx}$ satisfies the equation $9\frac{d^2y}{dx^2} - 3\frac{dy}{dx} - 2y = 0$ for all real values of x.

(4 marks)

4. (a) Find $\frac{d}{dx}(xe^x)$.

   (b) It is known that $\int_0^k xe^x \, dx = 1$. Find the value of k.

(4 marks)

5. (a) Find $\int \frac{x+2}{x^2+4x+9} dx$.

   (b) If $\frac{x^2+5x+11}{x^2+4x+9} \equiv A + \frac{Bx+C}{x^2+4x+9}$, find the values of the constants A, B and C.

   (c) Hence, find $\int \frac{x^2+5x+11}{x^2+4x+9} dx$.

(7 marks)

6. Define $f(x) = \frac{(\ln x)^2}{x}$ for all $x > 0$. Let α and β be two roots of the equation $f'(x) = 0$, where $\alpha > \beta$.

   (a) Express α in terms of e. Also, find β.

   (b) Using integration by substitution, evaluate $\int_\beta^\alpha f(x) dx$.

(7 marks)

7. An epidemic spreads in a town. There are N infected people t days after the outbreak of the epidemic. The rate of change of the number of the infected people can be modelled by

$$\frac{\mathrm{d}N}{\mathrm{d}t}=\frac{79200e^{-2t}}{(1+99e^{-2t})^2} \qquad (0 \le t \le 10).$$

It is given that $N = 100$ when $t = 0$.

(a) Express N in terms of t.

(b) Find the number of infected people one week after the outbreak of the epidemic, correct to the nearest integer.

(6 marks)

8. In Figure 1, the curve $y = x^2 - 4x + 5$ cuts the straight lines $y = 2x$ and $x = 3$ at the points A and B respectively.

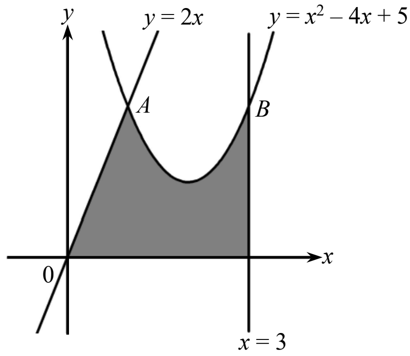

Figure 1

(a) Find the coordinates of A and B.

(b) Find the area of the shaded region bounded by the curve $y = x^2 - 4x + 5$, the x-axis and the lines $y = 2x$ and $x = 3$.

(6 marks)

9. A museum curator, Andrew, starts a promotion plan to boost the weekly number of visits to the museum. He models the weekly number of visits to the museum by

$$N = 10 \ln (t^2 - 4t + 15) + k,$$

where N is the weekly number of visits (in hundreds) recorded at the end of a week, t ($t \geq 0$) is the number of weeks elapsed since the start of the plan and k is a constant. Andrew finds that at the start of the plan (i.e. $t = 0$), the weekly number of visits is 20 hundreds.

(a) Find the exact value of k.

(b) How many weeks after the start of the plan will the weekly number of visits be the same as at the start of the plan?

(c) Find the minimum weekly number of visits.

(8 marks)

10. Let $f(x) = \dfrac{x^2 + ax + b}{x^2 + 1}$, where a and b are constants. The graph of $y = f(x)$ passes through $(1,2)$ and $f'(-1) = 4$.

(a) Find the values of a and b.

(b) Find the maximum and minimum point of the graph of $y = f(x)$.

(8 marks)

Section B (40 marks)

11. Consider the curve $C: y = \dfrac{x}{\sqrt{x-3}}$, where $x > 3$.

    (a) Find $\dfrac{dy}{dx}$.

    (2 marks)

    (b) A tangent to C passes through the point $(8, 0)$. Find the equation of this tangent.

    (6 marks)

12. Figure 2 shows an inverted right conical vessel. The base radius and the height of the vessel are 3 cm and 9 cm respectively. Water is poured into the vessel at a constant rate of 1 cm^3/s.

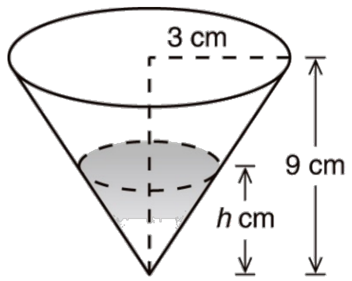

Figure 2

    (a) Let V cm^3 be the volume of water in the vessel when the depth of water is h cm. Express V in terms of h.

    (3 marks)

    (b) When the depth of water is 3 cm,

    (i) find the rate of change of the depth of water with respect to time.

    (ii) find the rate of change of the area of the water surface with respect to time.

    (5 marks)

13. An exhibition is held in a hall. On the first day of the exhibition, the number of visitors (in thousands) is modelled by

$$N = 7 - \frac{6}{1 + \alpha t e^{\beta t}},$$

where t ($0 \le t \le 12$) is the number of hours elapsed since the exhibition starts, α and β are constants.

(a) Express $\ln\left(\frac{N-1}{7t - Nt}\right)$ as a linear function of t.

(2 marks)

(b) It is given that the intercepts on the horizontal axis and the vertical axis of the graph of the linear function obtained in (a) are $\ln 5^{2.5}$ and $\ln 5$ respectively.

(i) Find α and β.

(ii) Find the maximum value of N.

(iii) Larry claims that the rate of change of N decreases from $t = 0$ to $t = 5$. Do you agree? Explain your answer.

(10 marks)

14. Let P(t) (in billion dollars) be the amount of the assets of a limited company at time t, where t is in years and $t \ge 0$. A financial analyst models the rate of change of P(t) as follows:

$$\mathrm{P}'(t) = \ln\left(\frac{t+6}{t+1}\right)$$

(a) Consider the interval $I = \int_0^4 \mathrm{P}'(t)\,\mathrm{d}t$.

(i) Using the trapezoidal rule with 4 subintervals, find an estimate of I.

(ii) It is known that P(0) = 5. Is it possible that the amount of the assets of the limited company will be more than 9.3 billion dollars at $t = 4$? Explain your answer.

(7 marks)

(b) Due to a financial crisis at $t = 4$, the limited company has a great loss in assets. Let Q(t) (in billion dollars) be the amount of the assets of the limited company at time t, where $t \ge 4$, and

$$\mathrm{Q}(t) = 6e^{\frac{-(t^2+3)}{2(t^2+4t-1)}}.$$

Is it possible that the amount of the assets of the limited company is more than 5 billion dollars? Explain your answer.

(5 marks)

End of Paper