Fukien Secondary School

S3 Final Examination (2020-2021)

Mathematics

(2 hours)

Date: 21st June 2021   Name: _____

Time: 8:30 a.m. – 10:30 a.m.   Class: _____ No. : ____

Instructions to students:

1.   This paper consists of THREE parts, Conventional Questions, Multiple-choice Questions and Bonus Question. There are Section A (1), Section A (2) and Section B in Conventional Questions. Section A (1) carries 35 marks. Section A (2) carries 29 marks. Section B carries 16 marks. Multiple-choice Questions carry 20 marks. Bonus Question carries 5 marks.

2.   The maximum score of this paper is 100.

3.   Attempt ALL questions in Conventional Questions and Multiple-choice Questions. Write your answers in the spaces provided in this Question / Answer Book.

4.   Unless otherwise specified, all workings must be clearly shown.

5.   Unless otherwise specified, numerical answers should either be exact or correct to 3 significant figures.

6.   The diagrams in this paper are not necessarily drawn to scale.

Conventional Questions

Section A (1) (35 marks)

1. Simplify $\dfrac{x^8}{(x^3 y^{-4})^2}$ and express the answers with positive indices.

(3 marks)

2. Make b the subject of the formula $a(a+b) = 2(b-a)$.

(3 marks)

3. Factorize
    (a) $a^2 + a - 6$,
    (b) $a^4 + a^3 - 6a^2$.

(3 marks)

4. (a) Solve the inequality $\dfrac{x+2}{-3} \le 9 - 4x$.

    (b) Write down all positive integers satisfying the inequality in (a).

(4 marks)

5. There were 250 students at a private school last year and the annual tuition fee of each student was $78 000. This year, the number of students decreased by 10% and the annual tuition fee of each student increased by 5%.

    (a) Find the total tuition fee received this year.

    (b) Find the percentage change in the total tuition fee received.

(4 marks)

6. In Figure 1, *PXQ* and *PYR* are straight lines.

    (a) Prove that $\triangle PXY \sim \triangle PQR$.
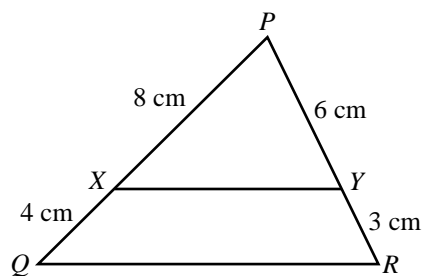
    (b) Hence, prove that *XY* // *QR*.

Figure 1

(5 marks)

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7. The following table shows the test results of Ben and Jane in three subjects.

	English	*Chinese*	*Mathematics*
Ben	60	81	90
Jane	70	75	78
Weight	3	4	2

    (a) Find the weighted mean marks of Ben and Jane.

    (b) Whose performance is better? Explain your answer.

(4 marks)

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8. Irene's pocket contains one $5 coin, two $2 coins and five $1 coins. If she draws a coin from her pocket randomly, find

    (a) the probabilities of drawing a $5 coin, a $2 coin and a $1 coin respectively,

    (b) the expected value of the coin drawn.

                                                                                                      (5 marks)

9. In Figure 2, the straight line L_1 passes through $A(8 , k)$ and O. The slope of the straight line L_2 is -4 and $L_1 \perp L_2$.

    (a) Find the slope of L_1.

    (b) Hence, find the value of k.
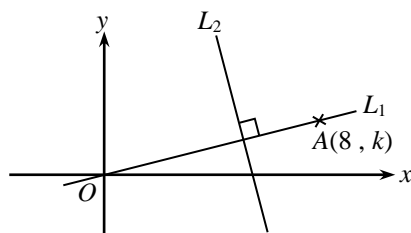
Figure 2

                                                                                                      (4 marks)

Section A (2) (29 marks)

10. In Figure 3, *ABCD* is a square and $\triangle ADE$ is an equilateral triangle. *BE* and *AD* intersect at *F*. Find $\angle AFE$.
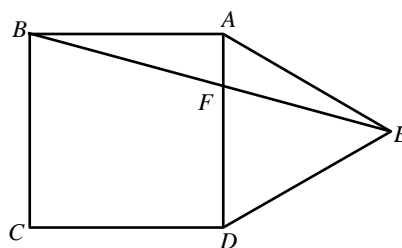
Figure 3

(4 marks)

11. Mable goes to office by one of the following means of transport: bus, minibus and taxi. The probability that she goes to office by bus, minibus and taxi are 0.3, 0.6 and 0.1 respectively. The table shows the fares that she goes to office by such means of transport.

Means of transport	Bus	Minibus	Taxi
Fare	$10.4	$16	$180

(a) Find the expected fare that Mable goes to office.

(b) A new MTR station is opened recently. Sometimes Mable goes to office by MTR instead of minibus. If the probability that Mable goes to office by MTR is x and the fare is $14.5, find the change in the expected fare in terms of x.

(5 marks)

12. In Figure 4, a student first walks from school (S) to a museum (M), and then to a bus stop (B). The distance between the school and the museum is 250 m and the distance between the museum and the bus stop is 400 m. It is known that M is at S50°E from S and B is due east of M.

(a) Find the distance between the school and the bus stop.

(b) Find the compass bearing of the bus stop from the school.
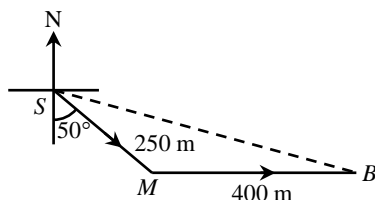
Figure 4

(6 marks)

13.   In Figure 5, the roly-poly toy consists of two spheres. The ratio of the surface areas of the two spheres is
      4 : 9.

      (a)   Find the diameter of the small sphere : the diameter of the large sphere.

      (b)   It is given that the volume of the toy is $\dfrac{140\pi}{3}$ cm³.

            (i)   Find the diameter of each sphere.

            (ii)   Christine wants to pack the toy in a box as a gift for a baby.
                   Do you think that a cylindrical box of height 11 cm and base
                   diameter 6.5 cm can be used as a gift box? Explain your answer.

Figure 5

(6 marks)

14. In Figure 6, $A(h, 1)$, $B(12, -2)$ and $C(2h, 4)$ are the three vertices of $\triangle ABC$. It is given that

$BC = 2AB$.

(a) Find the value of h.

(b) Is it true that $\angle ABC = 90°$? Explain your answer.

(c) Find the shortest distance between B and AC.

Figure 6

(8 marks)

Section B (16 marks)
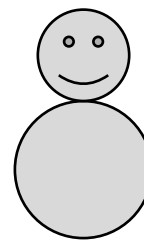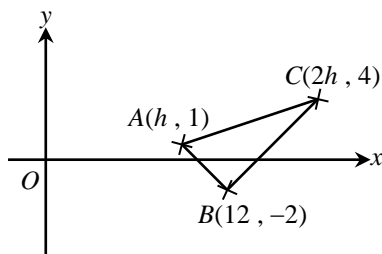
15. From the top of a building, the angle of depression of a cat on the ground is α. If the cat moves x m away from the building, the angle of depression becomes β.

    (a) Draw a diagram to show the situation.

    (b) If the height of the building is h m, show that $\dfrac{h}{x} = \dfrac{\tan \alpha \tan \beta}{\tan \alpha - \tan \beta}$.

    (c) If $h = \dfrac{2}{3}x$ and $\alpha + \beta = 90°$, prove that $2\tan^2 \alpha - 3\tan \alpha - 2 = 0$.

    (8 marks)

16. In Figure 7, *ABCD* and *BEFG* are squares. *BGC* is a straight line and *AG* is the angle bisector of $\angle CAB$. *AG* is produced to meet *CE* at *P*.

   (a)   Prove that $\triangle ABG \cong \triangle CBE$.

   (b)   Find $\angle ACP$ and $\angle AEP$.

   (c)   Hence, without using a calculator, express tan 22.5° in surd form.

Figure 7

(8 marks)

Multiple-choice Questions (20 marks)

Each question carries 2 marks. Put ✓ in the correct boxes.

	17	18	19	20	21	22	23	24	25	26
A										
B										
C										
D										

17. The lengths of 9 metal bars are shown below:

    21 cm, 27 cm, 21 cm, 29 cm, 23 cm, 21 cm, 29 cm, 30 cm, 24 cm

    Find the median of the lengths of the metal bars.

    A.   21 cm

    B.   23 cm

    C.   24 cm

    D.   25 cm

18. The value of a tablet computer depreciates by 20% every year. If the value of the tablet computer was $5 000 in 2015, find its value in 2020.

    A. $1 310.72

    B. $1 638.4

    C. $2 009.39

    D. $4 000

19. Refer to the Figure 8. Which of the following must be true?

    I. $\triangle PQR \cong \triangle RSP$

    II. $\triangle PQR \sim \triangle RSP$

    III. $PQ = RP$

    A.   I only

    B.   III only

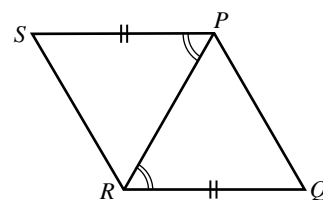   C.   I and II only

    D.   II and III only

Figure 8

20. Find the greatest integer satisfying $\dfrac{x}{4} - \dfrac{3-x}{5} \le -2$.

    A.   1

    B.   −3

    C.   −4

    D.   −28

21. In Figure 9, QS and RS are angle bisectors of $\triangle PQR$. Find $\angle QSR$.

    A.　110°

    B.　125°

    C.　140°

    D.　145°

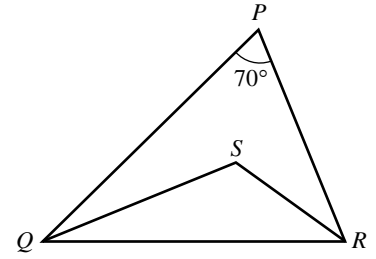Figure 9

22. How many planes of reflection does a cube have?

    A.　5

    B.　6

    C.　7

    D.　9

23. If a point C is randomly chosen from line segment AB, find the probability that $AC \le 3CB$.

    A.　$\dfrac{1}{4}$

    B.　$\dfrac{1}{3}$

    C.　$\dfrac{2}{3}$

    D.　$\dfrac{3}{4}$

24. In Figure 10, the slopes of the lines L_1, L_2 and L_3 are m_1, m_2 and m_3 respectively. Which of the following is true?

    A.　$m_1 > m_2 > m_3$

    B.　$m_1 > m_3 > m_2$
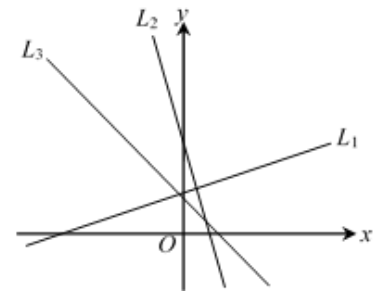
    C.　$m_3 > m_1 > m_2$

    D.　$m_3 > m_2 > m_1$

Figure 10

25. $P(4 , 4)$ is a point on the line segment AB such that $AP : PB = 2 : 3$. Given that the coordinates of A are $(-2 , 2)$, find the coordinates of B.

    A.　$(13 , 7)$

    B.　$(11 , 8)$

    C.　$(9 , 6)$

    D.　$(2 , 11)$

26. In Figure 11, *ACE*, *AGF* and *BDF* are straight lines. Find *CG* : *GD*.

    A.   1 : 2
    B.   1 : 3
    C.   2 : 5
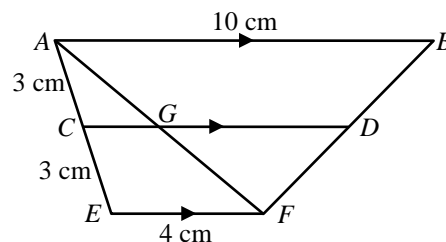   D.   3 : 5

Figure 11

Bonus Question (5 marks)

27. A point *P* is located inside a convex hexagon with perimeter 40 cm. 6 line segments are drawn from *P* to each vertex of the hexagon. Prove that the sum of the lengths of the 6 line segments drawn must be greater than 20 cm.

END OF PAPER