FUKIEN SECONDARY SCHOOL S5 First Term Examination (2020 – 2021) Mathematics Compulsory Part (2 hours)

Date: 8th January 2021 Time: 8:30 a.m. - 10:30 a.m.

Name:	
Class:	No.:

Instructions to students:

- This paper consists of FIVE sections. There are Section A(1), Section A(2), Section B, Multiple-choice Questions and Bonus Question.
- Attempt ALL questions in Section A(1), Section A(2), Section B and Multiple-choice Questions.
 Write your answers in the spaces provided in this Question / Answer Book.
- 3. Unless otherwise specified, show your workings clearly.
- 4. Unless otherwise specified, numerical answers should either be exact or correct to 3 significant figures.
- 5. The diagrams in this paper are not necessarily drawn to scale.

Section A(1) (34 marks)

- 1. Make *n* the subject of the formula $m = \frac{n+1}{n} 2$. (3 marks)
- 2. Factorize
 - (a) $p^2 9q^2$, (b) $p^2 - 9q^2 - (p - 3q)^2$.

(4 marks)

- 3. A shop owner sets the marked price of a handbag as \$650, which is 25% higher than its cost.
 - (a) Find the cost of the handbag.
 - (b) The shop owner claims that he should not offer a discount of more than 20% on its marked price, otherwise he will suffer a loss. Do you agree? Explain your answer.

(4 marks)

 S5 Mathematics Compulsory Part

4. It is given that *P* varies directly as \sqrt{Q} and inversely as *R*. When Q = 4 and R = -2, P = 3.

- (a) Express P in terms of Q and R.
- (b) Find the value of *P* when Q = 9 and R = 3.

(3 marks)

5.	The 6th term of a geometric sequence is 30, the 7th term is 45 and the last term is	$\frac{1215}{8}$. Find
	(a) the first term and the common ratio of the sequence,	
	(b) the number of terms of the sequence.	
		(5 marks)

- 6. In Figure 1, *O* is the centre. If $\angle OBA = 25^{\circ}$ and $\angle OCB = 55^{\circ}$, find
 - (a) $\angle ABC$,
 - (b) ∠*AOC*.





(4 marks)

7. Consider the arithmetic sequence 82, 76, 70,
(a) Find the general term *T_n* of the sequence.

(b) Find the first negative term of the sequence.

(5 marks)

- 8. In Figure 2, *EF*, *FG*, *GH* and *HE* are tangents to the circle at points *A*, *B*, *C* and *D* respectively. If $\angle CHD = 56^\circ$, $\angle BGC = 68^\circ$ and *EF* // *HG*, find
 - (a) $\angle BCD$,
 - (b) $\angle BAD$.



Figure 2

(6 marks)

Section A(2) (26 marks)

- 9. It is given that f(x) is the sum of two parts. One part varies directly as x and the other part varies directly as x^2 . Suppose f(1) = 5 and f(3) = 27.
 - (a) Find f(x). (3 marks)
 - (b) By the method of completing the square, find the maximum or minimum value of f(x) and the corresponding value of *x*. (5 marks)











- 11. Mable joins a savings plan. She deposits \$1 000 into her savings account at the beginning of each month. The interest rate is 1.5% per annum, compounded monthly.
 - (a) Find the amount in the account at the end of the 36th month correct to the nearest dollar.

(4 marks)

(b) Express, in terms of *n*, the amount in the account at the end of the *n*th month.

(2 marks)

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- 12. Let α and β be real numbers such that $\begin{cases} \beta = 28 3\alpha \\ \beta = -\alpha^2 + 5\alpha + 12 \end{cases}$.
 - (a) Find α and β . (3 marks)
 - (b) The 1st term and the 2nd term of an arithmetic sequence are $\log \alpha$ and $\log \beta$ respectively. Find the sum of the first 49 terms of the sequence.

(3 marks)

Section B (20 marks)

13. In Figure 4, *O* is the centre of the circle. *A*, *B*, *P* and *Q* are points on the circle. *M* is the mid-point of *AB.PC* and *QD* are the tangents to the circle at *P* and *Q* respectively. *CAMBD* and *PMQ* are straight lines.



Figure 4

(a)	Prove that <i>OMCP</i> is a cyclic quadrilateral.	(4 marks)
(b)	Prove that <i>OMQD</i> is a cyclic quadrilateral.	(3 marks)
(c)	Prove that $OC = OD$.	(3 marks)

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Figure 5

In Figure 5, OA_0B_0 is a right-angled triangle with $\angle A_0OB_0 = 45^\circ$ and $OA_0 = 100$ cm. A_1, A_2, A_3, \ldots , A_n, \ldots are points lying on OA_0 and $B_1, B_2, B_3, \ldots, B_n, \ldots$ are points lying on OB_0 such that A_1B_1 ,

 A_2B_2, A_3B_3, \dots are perpendicular to OA_0 and $\widehat{A_0B_1}, \widehat{A_1B_2}, \widehat{A_2B_3}, \dots$ are arcs with centre O.

- (a) (i) Find the exact lengths of A_0B_0 , A_1B_1 and A_2B_2 .
 - (ii) Show that the lengths of A_0B_0 , A_1B_1 , A_2B_2 , ... form a geometric sequence. (5 marks)
- (b) For $n \ge 1$, each region bounded by OB_0 , $A_{n-1}B_{n-1}$ and $A_{n-1}B_n$ is shaded.
 - (i) Find the exact area of the shaded region $A_0B_0B_1$.
 - (ii) Find the exact total area of all the shaded regions.

(5 marks)

Multiple-choice Questions (20 marks)

Each question carries 2 marks. Write down the correct answers in the boxes.

15.	16.	17.	18.	19.	20.	21.	22.	23.	24.

15. Suppose y varies directly as x^2 . When x = 3, y = 18. When x = 6, y =

A.
$$\frac{2}{9}$$
.

- **B.** $\frac{9}{2}$.
- **C.** $\frac{1}{72}$.
- **D**. 72.
- 16. Suppose y varies directly as \sqrt{x} . If x is decreased by 36%, find the percentage decrease in y.
 - **A.** 6%
 - **B**. 8%
 - **C.** 20%
 - **D.** 80%

17.



Figure 6

In Figure 6, *DE* is the tangent to the circle at *D*. Find $\angle CDE$.

- **A.** 52°
- **B.** 54°
- **C.** 70°
- **D.** 74°



Figure 7

In Figure 7, *O* is the centre of the circle *PQR*. *BC* is a diameter of the circle *ABOC* and *BC* is the tangent to the circle *PQR* at *P*. *APKR*, *BQOR* and *OKC* are straight lines. Which of the following must be true?

- I. *O*, *P*, *C* and *R* are concyclic.
- II. O, Q, P and K are concyclic.

III.*AB* // *PQ*

- IV. $\triangle CPK \sim \triangle OPQ$
- A. I and II only
- **B.** II and III only
- C. II and IV only
- **D.** I, III and IV only





Figrure 8

In Figure 8, *O* is the centre of the circle *QST*. *PR* is the tangent to the circle at *Q* and $\angle OTQ = 25^{\circ}$. If *QT* is the angle bisector of $\angle PQS$, then $\angle SQR =$

- **A.** 50°.
- **B.** 55°.
- **C.** 60°.
- **D.** 65°.

20.



Figure 9

In Figure 9, PQR is a circle. QP is produced to S such that RS is the tangent to the circle at R. I is the in-centre of \triangle PRS. If \angle ISP = 14° and \angle PRQ = 48°, then \angle PQR =

- **A.** 28°.
- **B.** 45°.
- **C.** 50°.
- **D.** 52°.
- 21. The sum of the first *k* terms of the arithmetic sequence -34, -28, -22, ... is 176. Find the value of *k*.
 - **A.** 14
 - **B.** 16
 - **C.** 18
 - **D.** 20
- 22. Let a_n be the *n*th term of a geometric sequence. If $a_{10} = 35$ and $a_{12} = 140$, which of the following must be true?
 - I. $a_1 > 0$
 - II. $a_8 < a_{10}$
 - III. $a_4 + a_6 + \ldots + a_{28} > 0$
 - A. I only
 - **B.** II only
 - C. I and III only
 - **D.** II and III only
- 23. The common ratio and the sum of the first 6 terms of a geometric sequence are $\frac{2}{3}$ and 2660 respectively. Find the first term of the sequence.
 - **A.** 768
 - **B.** 972
 - **C.** 1152
 - **D.** 1458

(4 marks)

24. S_1, S_2, S_3, \ldots are figures made up of identical regular octagons as shown below.



Find the total number of regular octagons in the first 8 figures.

- **A.** 126
- **B.** 144
- **C.** 180
- **D.** 288

Bonus Question (6 marks)

- 25. Elaine applies for a loan of \$2 000 000 from a bank at an interest rate of 4.5% per annum, compounded monthly. For each successive month after the day when the loan is taken, she pays a monthly instalment of \$15 000 until the loan is fully repaid. Note that the last instalment may be less than \$15 000.
 - (a) Suppose that the loan is not yet fully repaid after paying the *n*th instalment. Express the amount that Elaine still owes the bank after paying the *n*th instalment in terms of *n*.
 - (b) Find the number of months for Elaine to fully repay the loan. (2 marks)

END OF PAPER