FUKIEN SECONDARY SCHOOL
S5 First Term Examination (2020-2021)
Mathematics Extended Part Module 2
(2 hours)

Date: 12th January 2021 Name: ___________________

Time: 8:30 a.m.-10:30 a.m. Class: ________ No.: ______

Instructions to students:

1. This paper consists of TWO sections, A and B.
2. Attempt ALL questions in Section A and Section B.
3. Write your answers in the Answer Book provided.
4. Unless otherwise specified, show all workings clearly.
5. Unless otherwise specified, numerical answers must be exact.

FORMULAS FOR REFERENCE

$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$	$\sin A + \sin B = 2\sin\frac{A+B}{2}\cos\frac{A-B}{2}$
$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$	$\sin A - \sin B = 2\cos\frac{A+B}{2}\sin\frac{A-B}{2}$
$\tan(A \pm B) = \dfrac{\tan A \pm \tan B}{1 \mp \tan A \tan B}$	$\cos A + \cos B = 2\cos\frac{A+B}{2}\cos\frac{A-B}{2}$
$2\sin A \cos B = \sin(A+B) + \sin(A-B)$	$\cos A - \cos B = -2\sin\frac{A+B}{2}\sin\frac{A-B}{2}$
$2\cos A \cos B = \cos(A+B) + \cos(A-B)$	
$2\sin A \sin B = \cos(A-B) - \cos(A+B)$	

Section A (50 marks)

1. Evaluate

(a) $\int_0^3 (x+2)\sqrt{4-x}\,dx$ (b) $\int_0^{\frac{\pi}{6}} \cos^3 3x\,dx$ (c) $\int_{-3}^{3} \frac{1}{(x^2+3)^{\frac{3}{2}}}\,dx$ (d) $\int_{-\frac{\pi}{4}}^{\frac{\pi}{4}} (1+2x^3\cos^5 x)\,dx$

(13 marks)

2. It is given that $f(x)$ is an even function. If $\int_0^6 f(x)dx = a$ and $\int_{-6}^{6} [f(x)+3x^2]dx = 440$, find the value of a.

(3 marks)

3. (a) Using integration by parts, evaluate $\int_1^3 x^2 \ln x \, dx$.

   (b) Using substitution $u = \frac{1}{x}$ and (a), evaluate $\int_{\frac{1}{3}}^{1} \frac{\ln x}{x^4} dx$.

(4 marks)

4. (a) Evaluate $\int_{\ln 2}^{\ln 3} \frac{e^{-x}}{1-e^{-x}} dx$.

   (b) If $\frac{1}{u(u-1)} \equiv \frac{A}{u} + \frac{B}{u-1}$, find the values of A and B.

   (c) Using (a) and (b), or otherwise, evaluate $\int_{\ln 2}^{\ln 3} \frac{1}{e^x(e^x-1)} dx$.

(6 marks)

5. Figure 1 shows the curve $y^2 + 1 = 2(x+1)$. $P(4, 3)$ is a point on the curve.

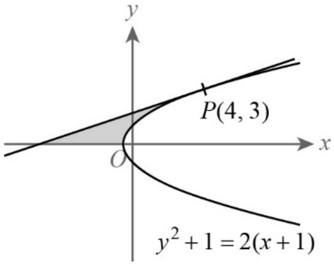

Figure 1

(a) Find the equation of the tangent to the curve at P.

(b) Find the area of the shaded region bounded by the curve, the tangent and the x-axis.

(6 marks)

6. (a) Prove that $\int_a^b f(x)dx = \int_a^b f(a+b-x)dx$.

(b) Hence, or otherwise, evaluate the following definite integrals.

(i) $\int_{\frac{\pi}{6}}^{\frac{\pi}{3}} \ln(\tan x)dx$

(ii) $\int_5^{10} \frac{x^{10}}{x^{10} + (15-x)^{10}} dx$

(6 marks)

7. In Figure 2, a cylinder is inscribed in a sphere of radius 2 cm.
Find the maximum volume of the cylinder.

(6 marks)

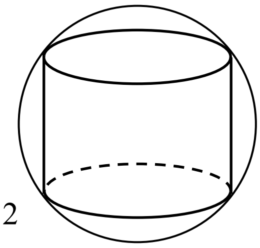

Figure 2

8. Consider the curve C: $y = 10e^{-x}$, where $x > 0$. It is given that P is a point lying on C. The horizontal line which passes through P cuts the y-axis at the point D, and the curve C cuts the y-axis at the point E. Denote the y-coordinate of P by v.

(a) Find the area of $\triangle PDE$ in terms of v.

(b) If P moves along C such that PD increases at a rate of $\frac{1}{5}$ units per second, find the rate of change of the area of $\triangle PDE$ when $PD = \ln 2$ units.

(6 marks)

Section B (50 marks)

9. Let $f(x)$ be a continuous function defined on R. Denote the curve $y = f(x)$ by Γ. It is given that the y-intercept of Γ is 1 and $f'(x) = 2x - 5$ for all $x \in \mathbf{R}$.

   (a) Find the equation of Γ.

   (b) Let L be a tangent to Γ such that L passes through the point $(3, -9)$ and the slope of L is positive. Denote the point of contact of Γ and L by P.

   (i) Find the coordinates of P.

   (ii) Find the equation of straight line which is perpendicular to L and passes through P.

(8 marks)

10. Let $f(x)=\dfrac{(x+2)^3}{(x-2)^2}$ for all real numbers $x \neq 2$. Denote the graph of $y=f(x)$ by Γ.

(a) Find the asymptote(s) of Γ.

(b) Find $f''(x)$.

(c) Find the maximum/minimum point(s) of Γ.

(d) Someone claims that there are two points of inflexion of Γ. Do you agree? Explain your answer.

(e) Find the area of the region bounded by Γ, the x-axis and the y-axis.

(14 marks)

11. (a) In Figure 3, the curve Γ consists of the curve AB, the line segments BC and CO, where O is the origin, B lies in the first quadrant and C lies on the x-axis. The equations of AB and BC are $x^2 - 3y + 5 = 0$ and $x + y - 5 = 0$ respectively.

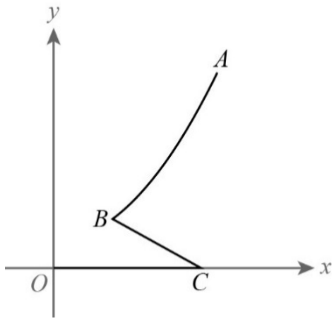

Figure 3

(i) Find the coordinates of B.

(ii) Let h be the y-coordinate of A, where $h > 3$. A cup is formed by revolving Γ about the y-axis. Prove that the capacity of the cup is $\frac{\pi}{2}(3h^2 - 10h + 81)$.

(b) A cup described in (a)(ii) is placed on a horizontal table. The radii of the base and the rim of the cup are 5 cm and 7 cm respectively.

(i) Find the capacity of the cup.

(ii) Water is poured into the cup at a constant rate of 20π cm^3/s. Find the rate of change of the depth of water when the volume of water in the cup is 53π cm^3.

(14 marks)

12. (a) Let $0 \le x \le \frac{\pi}{4}$. Prove that $\frac{1}{12\cos 2x + 13} = \frac{\sec^2 x}{24 + \sec^2 x}$.

(b) Evaluate $\int_0^{\frac{\pi}{4}} \frac{1}{12\cos 2x + 13} dx$.

(c) Let $s(x)$ be a continuous function defined on R such that the graph of $y = s(x)$ is symmetrical about the origin. Prove that $\int_{-t}^{t} s(x)\ln(1+e^x)dx = \int_0^t xs(x)dx$ for any $t \in \mathbf{R}$.

(d) Evaluate $\int_{-\frac{\pi}{4}}^{\frac{\pi}{4}} \frac{12\sin 2x}{(12\cos 2x + 13)^2} \ln(1+e^x)dx$.

(14 marks)

— End of Paper —