Fukien Secondary School

S4 First Term Examination (2020-2021)

Mathematics Compulsory Part

(2 hours)

Date: 7th January, 2021 Name: ___________________

Time: 8:30 a.m.- 10:30 a.m. Class: ________ No. : _____

Instructions to students:

1. This paper consists of three parts, Conventional Questions, Multiple-choice Questions and Bonus Question. There are Section A(1), Section A(2) and Section B in Conventional Questions. Section A(1) carries 34 marks.
Section A(2) carries 27 marks. Section B carries 19 marks. Multiple-choice Questions carry 20 marks. Bonus carries 5 marks.

2. The maximum score of this paper is 100.

3. Attempt ALL questions in Conventional Questions and Multiple-choice Questions. Write your answers in the spaces provided in this Question / Answer Book.

4. Unless otherwise specified, show all workings clearly.

5. Unless otherwise specified, numerical answers should either be exact or correct to 3 significant figures.

6. The diagrams in this paper are not necessarily drawn to scale.

Conventional Questions

Section A(1) (34 marks)

1. Make y the subject of the formula $x = \dfrac{5y-1}{1+y}$.

(3 marks)

2. Factorize
   (a) $24x - 2x^2$,
   (b) $x^2 - 13x + 12$,
   (c) $x^2 - 13x + 12 + 24x - 2x^2$.

(4 marks)

3. There are 125 boys in S3. If 60% of the number of girls is equal to 48% of the number of boys,
   (a) find the number of girls in S3,
   (b) find the total number of students in S3.

(4 marks)

4. Solve the equation $(x+1)(x-1)=(2-x)(1+x)$.

(3 marks)

5. If the graph of $y = 3x^2 + 2x + 4k$ does not intersect the x-axis, find the range of values of k.

(3 marks)

6. If $f(x) = 2x + 3$, $g(x) = x^2 + 3$ and $h(x) = \dfrac{g(x)}{f(x)}$, find

(a) the value of $f(1) \bullet g(-1)$,

(b) the domain of $h(x)$.

(3 marks)

7. It is given that there is exactly one x-intercept on the graph of a quadratic function $y = f(x)$. The x-intercept and y-intercept of the graph are 2 and 3 respectively.

   (a) Find the coordinates of vertex on the graph of $y = f(x)$.

   (b) Find $f(x)$.

(4 marks)

8. Let p be a non-zero constant. When $f(x) = x^3 - px^2 + 2px - 3p$ is divided by $x - p$, the remainder is $5p$. Find

   (a) p,

   (b) the remainder when $f(x)$ is divided by x.

(5 marks)

9. Let c and d be non-zero real numbers.
   (a) Simplify $4(c + di)(2 + 5i)$ and express the answer in the form $a + bi$.
   (b) If $4(c + di)(2 + 5i)$ is a real number, find $c : d$.

(5 marks)

Section A(2) (27 marks)

10. It is given that the vertex of the graph of a quadratic function $y = f(x)$ is $A(2 , -3)$. If the graph intersects the x-axis at the origin O and at another point B,
    (a) find the coordinates of point B,
    (b) find the area of $\triangle OAB$.

(6 marks)

11. It is given that the quadratic equation $9x^2 - kx + 1 = x$ has two equal real roots.
    (a) Find the two possible values of the constant k.
    (b) If k takes the negative value obtained, solve the equation.

(6 marks)

12. $f(x)$ is a polynomial. When $f(x)$ is divided by $x - 2$ and $x + 1$, the remainders are -8 and 10 respectively. When $f(x)$ is divided by $x^2 - 5x + 4$, the remainder is 0 and the quotient is $ax + b$.

(a) Find the values of a and b.

(b) Solve $f(x) = 0$.

(7 marks)

13. In Figure 1, ABC is a triangle, where $AB = BC = CA$, $DM = 2$ cm. $DEFG$ is a rectangle, and D, E, F, G, M are points on the sides of $\triangle ABC$. Suppose $BD = x$ cm, $BG = 2x$ cm. It is given that the area of $\triangle ABC$ is twice that of $DEFG$.

    (a) Express DG in terms of x.

    (b) Express AM in terms of x.

    (c) Find x.

(8 marks)

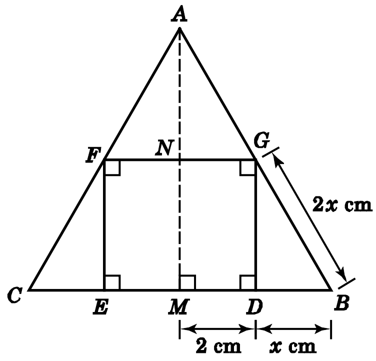

Figure 1

Section B (7-19 marks)

In this section, answer part (X) or part (Y) in each question. Do not answer both parts. If both parts in a question are attempted, only part (X) will be marked.

14X. It is given that α and β are the roots of the quadratic equation $2x^2 + k = 2(2-k)x$, where k is a real number.

(a) Express $\alpha + \beta$ and $\alpha\beta$ in terms of k.

(2 marks)

(b) It is given that $(\alpha - \beta)^2 = 76$. Find the value(s) of k.

(3 marks)

OR

14Y. Suppose α and β are the roots of equation $2x^2 + 8x - 9 = 0$, find the value of $\frac{1}{\alpha} + \frac{1}{\beta}$ without solving the equation.

(2 marks)

15X. Let R_1 and R_2 be the remainders when polynomials $f(x)$ and $g(x)$ are divided by $2x + 1$ respectively. It is given that $R_1 = kR_2$, where k is a non-zero constant.

(a) Show that $f(x) - kg(x)$ is divisible by $2x + 1$. (2 marks)

(b) Consider $f(x) = 4x^4 - 7x^3 - 10x^2 + 11x + 8$ and $g(x) = x^4 + 2x^3 - x^2 - 24x - 11$.

(i) Find the remainders when $f(x)$ and $g(x)$ are divided by $2x + 1$ respectively.

(ii) Hence, find a degree four polynomial $h(x)$ such that $2x + 1$ is one of its factors. (5 marks)

OR

15Y. Find the quotient and the remainder of $(9x^2 + 27x^3 - 3x - 35) \div (3x - 2)$. (2 marks)

16X. In Figure 2, a fence *PQRST* of length 120 m is used to surround the L-shaped region outside the rectangular building.
It is known that $PQ = TS = x$ m, $QR = RS$ and the area of the L-shaded region *PQRSTU* is N m^2.

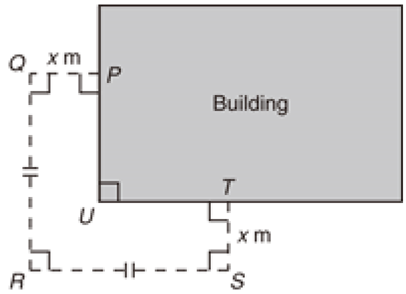

Figure 2

(a) Express the lengths of QR and PU in terms of x.

(2 marks)

(b) Express N in terms of x.

(2 marks)

(c) Using the method of completing the square, find the maximum value of N.

(3 marks)

Or

16Y. Using the method of completing the square, find the coordinates of the vertex of the graph of $y = 2x^2 - 40x + 6$.

(3 marks)

Multiple-choice Questions (20 marks)

Write down the best answer to each question in the corresponding box.

17	18	19	20	21	22	23	24	25	26

17. Which of the following quadratic equations can be formed with R and $-S$ as roots?

    A. $x^2 - (R + S)x + RS = 0$

    B. $x^2 + (R - S)x - RS = 0$

    C. $x^2 + (S - R)x - RS = 0$

    D. $x^2 + (R + S)x + RS = 0$

18. In Figure 3, O is the origin and the graph of $y = x^2 - 4x + 3$ intersects the x-axis at two points A and B. Find the length of $OA + OB$.

    A. 3

    B. 4

    C. 5

    D. 6

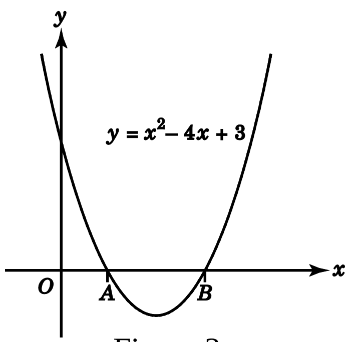

Figure 3

19. If a and b are rational numbers and c is irrational number, which of the following must be true?

    I. $a + b + c$ must be irrational.

    II. abc must be irrational.

    III. The roots of $ax^2 + bx + c = 0$ must be irrational if $a \neq 0$.

    A. I only

    B. I and II only

    C. II and III only

    D. I and III only

20. If a, b are distinct real numbers, and $\begin{cases} a^2 + 6a + 3 = 0 \\ b^2 + 6b + 3 = 0 \end{cases}$, find $a^2 + b^2$.

    A. 30

    B. 35

    C. 40

    D. 45

21. $(3i)^4 \times \left(\dfrac{-1}{3}i\right)^3 =$

A. 0
B. $3i$
C. $-3i$
D. $\dfrac{i}{3}$

22. If $g(x) = x^2 - 6x + 5$, then $3g(x) - g(3x) =$

A. 0
B. $6x^2 - 10$
C. $-6x^2 + 10$
D. $12x^2 + 20$

23. Figure 4 shows the graph of $y = ax^2 + bx + c$. Which of the following is correct?

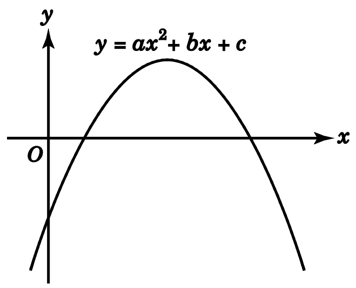

Figure 4

A. $a > 0$, $c > 0$ and $b^2 - 4ac < 0$
B. $a > 0$, $c < 0$ and $b^2 - 4ac < 0$
C. $a < 0$, $c > 0$ and $b^2 - 4ac > 0$
D. $a < 0$, $c < 0$ and $b^2 - 4ac > 0$

24. Given that $f(x) = 6x^3 + 41x^2 - 9x - 14$ and $f(-7) = 0$, factorize $f(x)$.

A. $(x + 7)(2x + 1)(3x - 2)$
B. $(x - 7)(2x - 1)(3x + 2)$
C. $(x - 7)(2x + 1)(3x + 2)$
D. $(x + 7)(2x - 1)(3x - 2)$

25. Find the H.C.F. of $(x + 1)^2(x - 4)(x + 3)$ and $5(x + 1)(x - 4)^2$.

A. $5(x + 1)^2(x - 4)^2(x + 3)$

B. $(x + 1)(x - 4)(x + 3)$

C. $(x + 1)(x - 4)$

D. $5(x + 1)(x - 4)(x + 3)$

26. Simplify $\frac{1}{a^2 - 4} + \frac{2a}{a^2 - 6a + 8} - \frac{2}{a^2 - 2a - 8}$.

A. $\frac{2a^2 + a + 5}{(a+2)(a-2)(a-4)}$

B. $\frac{a(2a+3)}{(a+2)(a-2)(a-4)}$

C. $\frac{2a+3}{(a+2)(a-2)(a-4)}$

D. $\frac{1}{(a-2)(a-4)}$

Bonus Question (5 marks)

27. Let n be a positive integer such that $n^{2018}-1$ is divisible by $(n-1)^2$.
Find the sum of all possible values of n.
Hint: Find $Q(n)$ in $n^{2018}-1=(n-1)Q(n)$ first.

(5 marks)

- End of Paper -