FUKIEN SECONDARY SCHOOL

S4 First Term Examination (2020-2021)

Mathematics Extended Part Module 2

(2 hours)

Date: 15th January 2021 Name:___________________

Time: 8:30 a.m.-10:30 a.m. Class: ________ No.: ______

**Instructions to students**:

1. This paper consists of TWO sections, A and B.
2. Attempt ALL questions in Section A and Section B.
3. Write your answers in the Answer Book provided.
4. Unless otherwise specified, show all workings clearly.
5. Unless otherwise specified, numerical answers must be exact.

**FORMULAS FOR REFERENCE**

| | |
|---|---|
| $\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$ | $\sin A + \sin B = 2\sin\frac{A+B}{2}\cos\frac{A-B}{2}$ |
| $\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$ | $\sin A - \sin B = 2\cos\frac{A+B}{2}\sin\frac{A-B}{2}$ |
| $\tan(A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A \tan B}$ | $\cos A + \cos B = 2\cos\frac{A+B}{2}\cos\frac{A-B}{2}$ |
| $2\sin A \cos B = \sin(A+B) + \sin(A-B)$ | $\cos A - \cos B = -2\sin\frac{A+B}{2}\sin\frac{A-B}{2}$ |
| $2\cos A \cos B = \cos(A+B) + \cos(A-B)$ | |
| $2\sin A \sin B = \cos(A-B) - \cos(A+B)$ | |

**Section A (70 marks)**

1. (a) Prove that $\tan m\theta + \tan n\theta = \dfrac{\sin(m+n)\theta}{\cos m\theta \cos n\theta}$.

(b) Solve the equation $\tan\theta + \tan 2\theta = 0$ for $0 \le \theta \le \dfrac{\pi}{2}$.

(5 marks)

2. Prove, by mathematical induction, that for all positive integers $n$,

$$\sin x\cos 2x + \sin x\cos 4x + \sin x\cos 6x + \cdots + \sin x\cos 2nx = \frac{1}{2}\sin(2n+1)x - \frac{1}{2}\sin x.$$

(5 marks)

3. (a) Expand $\left(2-\dfrac{3}{x}\right)^5$.

(b) Find the coefficient of $x$ in the expansion of $(4+x)^3\left(2-\dfrac{3}{x}\right)^5$.

(5 marks)

4. It is given that the coefficient of $x^2$ in the expansion of $\left[1+\dfrac{2x}{n(n-1)}\right]^n$ is $\dfrac{1}{6}$, where $n$ is an integer greater than 1.

(a) Find the value of $n$.

(b) Find the coefficient of $x^3$.

(6 marks)

5. (a) Using mathematical induction, prove that $\sum_{k=1}^{n} \frac{1}{(2k-1)(2k+1)} = \frac{n}{2n+1}$ for all positive integers $n$.

(b) Using (a), evaluate $\sum_{k=101}^{200} \frac{80\ 601}{(2k-1)(2k+1)}$ .

(8 marks)

6. (a) Show that $t + 1$ is a factor of $t^3 - 3t^2 - 3t + 1$.

(b) Express $\tan 3x$ in terms of $\tan x$.

(c) Using the results of (a) and (b), show that $\tan \frac{5\pi}{12} = 2 + \sqrt{3}$ .

(8 marks)

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7. (a) Prove that $\sin 3\theta = 3\sin\theta - 4\sin^3\theta$.

In Figure 1, $OAB$ is a triangle with $AB = 1$, $\angle OAB = \theta$ and $\angle OBA = 3\theta$. $OY$ is an angle bisector of $\triangle OAB$ with $AY = x$ and $YB = y$.

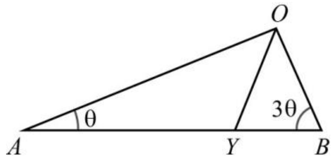



Figure 1

(b) Show that $\dfrac{x}{y} = 3 - 4\sin^2\theta$.

(c) It is given that $0° < \theta < 45°$. Find the range of values of $\dfrac{x}{y}$.

(8 marks)

8. (a) Prove that $\dfrac{\sin A\cos B + \cos A\sin B}{\cos A\cos B - \sin A\sin B} = \tan(A+B)$.

(b) Without using a calculator, find the value of $\dfrac{\sin 38°\cos 37° + \cos 38°\sin 37°}{\cos 38°\cos 37° - \sin 38°\sin 37°}$.

(8 marks)

9. (a) Express the expansions of $(3-x)^n$ and $(-1+x)^n$ in ascending powers of $x$ in summation notation, where $n$ is a positive integer.

(b) Prove that $(3-x)^n+(-1+x)^n=\sum_{r=0}^{n}C_r^n(-1)^r[3^{n-r}+(-1)^n]x^r$.

(c) Hence find $\sum_{r=0}^{100}C_r^{100}(-2)^r(3^{100-r}+1)$.

(8 marks)

10. It is given that $\sin\alpha=\frac{4}{5}$, $\tan\alpha<0$ and $\cot\beta=-2$. Find the value of each of the following.

(a) $\operatorname{cosec} 2\alpha$

(b) $\tan(\alpha-2\beta)$

(9 marks)

## Section B (30 marks)

11. (a) Prove, by mathematical induction, that for all positive integers $n$,

$$\sum_{r=1}^{n}(-1)^{r+1}\sin rx=\frac{\sin\dfrac{x}{2}+(-1)^{n+1}\sin\left(n+\dfrac{1}{2}\right)x}{2\cos\dfrac{x}{2}}\text{, where }\cos\frac{x}{2}\neq 0.$$

(b) Hence prove that $\displaystyle\sum_{r=10}^{33}(-1)^{r+1}\sin\frac{r\pi}{7}=\sin\frac{2\pi}{7}$.

(9 marks)

12. (a) Prove, by mathematical induction, that for all positive integers $n$, $\sum_{r=1}^{n} \frac{r+3}{2^r} = 5 - \frac{n+5}{2^n}$.

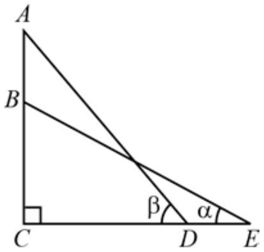
(b) Hence simplify $\frac{2n+4}{2^{2n+1}} + \frac{2n+5}{2^{2n+2}} + \frac{2n+6}{2^{2n+3}} + \cdots + \frac{3(n+1)}{8^n}$.

(c) It is given that $\sum_{i=1}^{n} \frac{1}{2^i} = 1 - \frac{1}{2^n}$ for all positive integers $n$.

Using the result of (a), simplify $\sum_{j=1}^{n} \frac{j}{2^j}$.

(12 marks)

13. In Figure 2, *ABC* and *CDE* are straight lines. It is given that $\angle BEC = \alpha$, $\angle ADC = \beta$, $AD = BE = y$ and $AC \perp CE$.




(a) (i) Prove that $DE = 2y\sin\left(\frac{\alpha+\beta}{2}\right)\sin\left(\frac{\beta-\alpha}{2}\right)$.

(ii) Prove that $AB = 2y\cos\left(\frac{\alpha+\beta}{2}\right)\sin\left(\frac{\beta-\alpha}{2}\right)$.

(b) Hence prove that $DE = AB\tan\left(\frac{\alpha+\beta}{2}\right)$.

(9 marks)

— End of Paper —