FUKIEN SECONDARY SCHOOL S4 First Term Examination (2020-2021) Mathematics Extended Part Module 1 (2 hours)

Date: 14th January 2021 Time: 8:30 a.m. – 10:30 a.m.

Name:	
Class:	No.:

Instructions

- 1. This paper consists of Section A (68 marks) and Section B(32 marks). Answer ALL questions in this paper.
- 2. Write your answers in the Answer Book.
- 3. Unless otherwise specified, all workings must be clearly shown.
- 4. Unless otherwise specified, numerical answers should be either exact or given to 4 decimal places.
- 5. The diagrams in this paper are not necessarily drawn to scale.

Section A (68 marks)

- 1. Jimmy invests \$10 000 in a bank at an interest rate of r% per annum. If the interest is compounded annually, the amount after 3 years is \$13 310.
 - (a) Find r.
 - (b) Find the amount if the interest is compounded continuously, correct your answers to the nearest dollar.

2. (a) Simplify
$$\frac{(n+2)!}{2!n!}$$
.

(b) Hence, solve $6C_2^{n+2} + C_1^{n-1} = 478$.

(6 marks)

3. Given that in the expansion of $\left(x^4 + \frac{1}{2x^2}\right)^n$ in descending powers of *x*, where *n* is a positive integer, the 9th term is the constant term. Find *n* and the constant term.

(4 marks)

- 4. (a) Expand e^{-2x} in ascending powers of x as far as the term in x^2 .
 - (b) The coefficient of x in the expansion of $\frac{(1+ax)^8}{e^{2x}}$ is 2 and $a \neq 0$.
 - (i) Find a.
 - (ii) Find the coefficient of x^2 .

(7 marks)

- 5. Solve each of the following equations.
 - (a) $\ln(5x-3) \ln(7x-19) = \ln 6$.
 - (b) $\ln x + \ln(x-6) = \ln 16$.
 - (c) $8e^x e^{-x} = 7$.

(9 marks)

- 6. Initially, the pH value of the water in a lake is 6. The lake is contaminated due to the leakage of the alkaline waste. A researcher then records daily the pH value of the water in the lake and models its pH value by $P(t) = a + t e^{-kt}$, where $t (\ge 0)$ is the time measured in days, *a* and *k* are positive constants.
 - (a) Find the value of *a*.
 - (b) It is found that P(2) P(1) = 0.228338. Find the value of k correct to 1 decimal place.

(6 marks)

(a)
$$\lim_{x \to 1} \frac{6x^2 - 7x + 1}{3x^2 - 2x - 1}$$

(b)
$$\lim_{x \to \infty} \frac{6x^2 - 7x + 1}{3x^2 - 2x - 1}$$

(c)
$$\lim_{x \to \infty} \frac{13 - 2e^x}{6e^{-2x} + 8e^x}$$

(d)
$$\lim_{x \to \infty} \frac{3x+1}{\sqrt{4x^2+7x-5}}$$

(8 marks)

8. (a) Expand
$$(\sqrt{x + \Delta x} - \sqrt{x})(\sqrt{x + \Delta x} + \sqrt{x})$$
.

(b) It is given that
$$y = f(x) = \sqrt{x}$$
.

(i) Find
$$\lim_{\Delta x \to 0} \frac{\Delta y}{\Delta x}$$
.
(ii) Hence find f

(ii) Hence, find f'(9).

(7 marks)

9. Differentiate each of the following functions with respect to x.

(a) $y = x^3 + 3x + 4 + 2x^{-2}$

(b)
$$y = (3x^2 + 2x + 1)^7$$

(c)
$$y = x^2 e^{-x}$$

$$(d) \quad y = x^2 \ln x$$

(e) $y = \ln(x^7 + 4x^2 + 100)$

(11 marks)

- 10. It is given that $y = e^{kx}$.
 - (a) Find $\frac{dy}{dx}$ and $\frac{d^2y}{dx^2}$.

(b) Hence, find the value(s) of k such that $\frac{d^2y}{dx^2} - 3\frac{dy}{dx} + 2y = 0$.

(6 marks)

Section B (32 marks)

- 11. It is given that f(-3) = -4 and f'(-3) = 2. Let $g(x) = \frac{6x-1}{f(x)}$. Find g'(-3).
- 12. The curve $y = f(x) = \frac{4x}{x^2 3}$ passes through the point A(a, 2) where a < 0.
 - (a) Find a.
 - (b) Find the equation of the tangent to the curve y = f(x) at the point A.

(8 marks)

(6 marks)

- 13. (a) Expand $(1 + ax)^2(1 + bx)^{10}$ in ascending powers of x as far as the term in x^2 .
 - (b) If the coefficients of x and x^2 in the expansion in (a) are 1 and $\frac{-19}{4}$ respectively,
 - (i) find the values of a and b,
 - (ii) find the coefficient of x in the expansion of $(1 + ax)^3(1 + bx)^{10}$ where b > 0.

(10 marks)

14. An ecologist studies the birds at Mai Po Nature Reserve. Only 21% of the birds are "residents", i.e. found throughout the year. The remaining birds are migrants. The ecologist suggests that the number N(t) of a certain species of migrants can be modelled by the function

$$N(t) = \frac{3000}{1 + ae^{-bt}},$$

where *a*, *b* are positive constants and *t* is the number of days elapsed since the first one of that species of migrants was found at Mai Po in that year.

- (a) Express $\ln \frac{1}{2} \frac{1}{2} \frac{1}{2}$ as a linear function of *t*.
- (b) It is given that the intercepts of the horizontal axis and the vertical axis of the graph of the linear function obtained in (a) are 13 and 3.9 respectively. Find the values of *a* and *b*, correct to 1 decimal place.
- (c) Using the values of a and b obtained in (b), find N(16).

(8 marks)

End of Paper